$$y'' + y = 0$$

$$\frac{d^2y}{dx^2} + y = 0$$

Find $y = y(x)$

$$e^{rt}(r^2+1) = 0$$

$$r^2 + 1 = 0$$

$y = e^{rt}$

$$r^2 = -1$$

$$r = \pm i$$

$$y = C_1 e^{it} + C_2 e^{-it}$$

$$y'' + 3y' + 2y = 0$$

$y = e^{rt}$

$y' = re^{rt}$

$y'' = r^2 e^{rt}$

$$r^2 e^{rt} + 3re^{rt} + 2e^{rt} = 0$$

$$\underline{\underline{e^{rt}}}\,(\underline{\underline{r^2+3r+2}}) = 0$$

$$(r+2)(r+1) = 0$$

$$r = -2, -1$$

$$y = C_1 e^{-t} + C_2 e^{-2t}$$

$$r^2 = 9$$

$$r = \pm 3$$

$y'' + y = 0$ $\quad y = a_0 + a_1 x + a_2 x^2 + \cdots$

$y = a_0 \underbrace{(\qquad + \cdots)}_{\cos x} + a_1 \underbrace{(\qquad + \cdots)}_{\sin x}$

$y'' = -y$ $\quad y$ is $\sin x$ or $\cos x$

$$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \cdots$$

$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \cdots$$

$$\cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \cdots$$

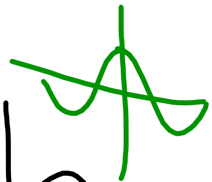
$$(-x)^3 = (-x)(-x)(-x) = -x^3$$

$$\sin(-x) = (-x) - \frac{1}{3!}(-x)^3 + \frac{1}{5!}(-x)^5 - \cdots$$

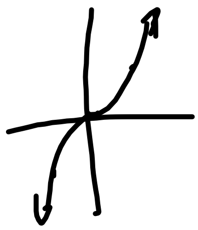
$$= -x + \frac{1}{3!}x^3 - \frac{1}{5!}x^5 + \cdots$$

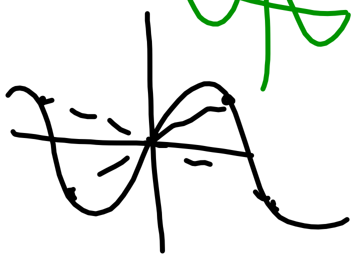
$$\sin(-x) = -\sin x$$

$$\cos(-x) = \cos(x)$$



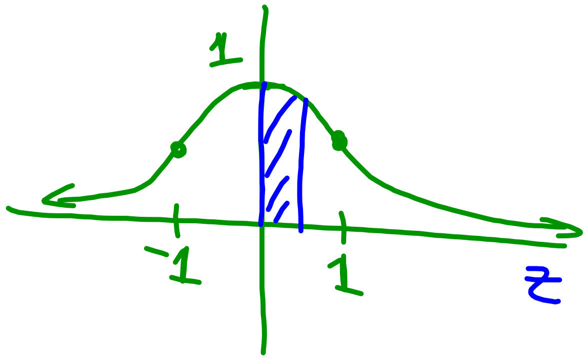
odd $\quad f(-x) = -f(x)$





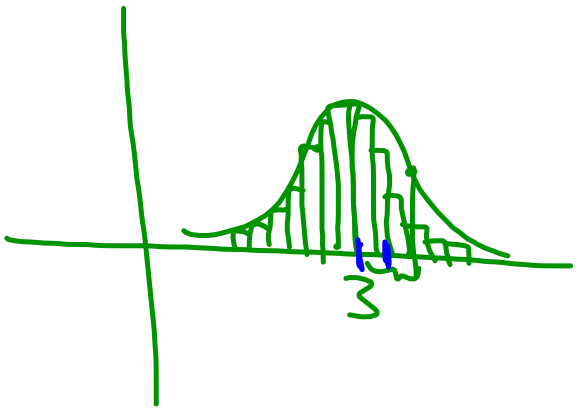
$$y = e^{-\frac{x^2}{2}}$$

$$\int_{-\infty}^{\infty} e^{-\frac{x^2}{2}}\,dx = \sqrt{2\pi}$$



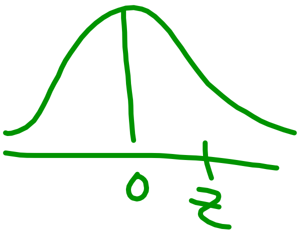
$$y = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$ normal distribution

"Gaussian"


3

$$P(0<Z<\underbrace{0.7}_{z}) = \frac{1}{\sqrt{2\pi}}\int_0^{0.7} e^{-\frac{x^2}{2}}\,dx$$

$$F(z) = \frac{1}{\sqrt{2\pi}}\int_0^{z} e^{-\frac{x^2}{2}}\,dx = \frac{1}{\sqrt{2\pi}}\int_0^{z}\left[\sum_{n=0}^{\infty}\frac{(-1)^n}{2^n n!}x^{2n}\right]dx$$



$$= \frac{1}{\sqrt{2\pi}}\sum_{n=0}^{\infty}\left[\int_0^{z}\frac{(-1)^n}{2^n n!}x^{2n}\,dx\right]$$

$$= \frac{1}{\sqrt{2\pi}}\sum_{n=0}^{\infty}\frac{(-1)^n}{2^n n!}\int_0^{z}x^{2n}\,dx$$

$$= \frac{1}{\sqrt{2\pi}}\sum_{n=0}^{\infty}\frac{(-1)^n}{2^n n!(2n+1)}x^{2n+1}$$

$$F(z) = \frac{1}{\sqrt{2\pi}}\left(x - \frac{1}{6}x^3 + \frac{1}{40}x^5 - \frac{1}{336}x^7 + \cdots\right)$$

$$F(0.7) \approx 0.2580$$