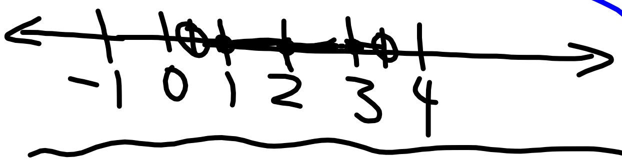
Give the interval notation for the set of numbers x that satisfy

$|x-2| < \frac{3}{2}$

$|a-b|$ is dist btwn a and b



$\left(2-\frac{3}{2}, 2+\frac{3}{2}\right) = \left(\frac{1}{2}, \frac{7}{2}\right)$

<u>Exam 2:</u> Tuesday, May 9th

$$\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$

<u>Power Series</u> (centered at zero)

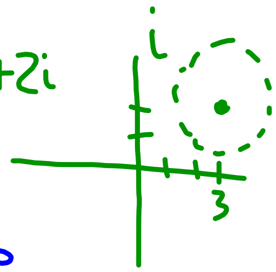
$$\sum_{n=0}^{\infty} a_n (x-c)^n = a_0 + a_1(x-c) + a_2(x-c)^2 + \cdots$$

Power Series centered at c

Suppose $\sum_{n=0}^{\infty} a_n (x-c)^n$ converges for

x in $\underbrace{(c-R, c+R)}_{\text{Interval of convergence}}$

3+2i



R is radius of convergence

$$\sum_{n=0}^{\infty} a_n(x-c)^n \quad \text{and} \quad \sum_{n=0}^{\infty} |a_n(x-c)|$$

have same radius + interval of convergence

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

Radius + interval of convergence?

Look at $\sum_{n=0}^{\infty} \left|\frac{x^n}{n!}\right|$, think of $\left|\frac{x^n}{n!}\right|$ as a_n for ratio test

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{\left|\frac{x^{n+1}}{(n+1)!}\right|}{\left|\frac{x^n}{n!}\right|} = \lim_{n\to\infty} \left|\frac{x^{n+1}}{(n+1)!} \cdot \frac{n!}{x^n}\right| = \underbrace{\lim_{n\to\infty} \left|\frac{x}{n+1}\right| = 0 < 1}_{\text{regardless of what } x \text{ is.}}$$

Series converges for all x

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$$ (center is zero)

$|ab| = |a||b|$

$$\lim_{n\to\infty} \frac{\left|(-1)^{n+2} \frac{x^{n+1}}{n+1}\right|}{\left|(-1)^{n+1} \frac{x^n}{n}\right|} = \lim_{n\to\infty} \left| \frac{x^{n+1}}{n+1} \cdot \frac{n}{x^n} \right|$$

$$= \lim_{n\to\infty} \left|\frac{n}{n+1}\right| |x| = |x| < 1$$

1

The series converges on $(-1,1)$

Radius of convergence is $\underline{1}$

We don't $\underline{\text{yet}}$ know what happens at 1 and -1.

$$\sum_{n=0}^{\infty} 2^n (x-3)^n$$

$$\lim_{n\to\infty} \frac{|2^{n+1}(x-3)^{n+1}|}{|2^n(x-3)^n|} = \lim_{n\to\infty} |2(x-3)| = 2|x-3| < 1$$

$$|x-3| < \tfrac{1}{2}$$

Center: 3 Radius: $\frac{1}{2}$

$$|x-c| < R$$

Interval: $(2\frac{1}{2}, 3\frac{1}{2})$

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{n} \qquad \text{Interval: } (-1,1)$$

$$x=1: \sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n} \quad \text{converges}$$

$$x=-1: \sum_{n=1}^{\infty}(-1)^{n+1}\frac{(-1)^n}{n} = \sum_{n=1}^{\infty}\frac{(-1)^{2n+1}}{n} = -\sum_{n=1}^{\infty}\frac{1}{n} \quad \text{diverges}$$

Converges on $[-1,1]$

$$\sum_{n=0}^{\infty} 2^n (x-3)^n$$

Interval: $\left(2\tfrac{1}{2}, 3\tfrac{1}{2}\right)$

$x = 2\tfrac{1}{2}$: $\displaystyle\sum_{n=0}^{\infty} 2^n\left(-\tfrac{1}{2}\right)^n = \sum_{n=0}^{\infty}\left[2\left(-\tfrac{1}{2}\right)\right]^n = \sum_{n=0}^{\infty}(-1)^n$

$x = 3\tfrac{1}{2}$: