If $y = \sum_{n=0}^{\infty} a_n x^n$, what are y' and y''?

$5x^7 \rightarrow 35x^6$

$y = a_0 + a_1 x + a_2 x^2 + \cdots$

$$y' = \sum_{n=1}^{\infty} n a_n x^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$$

$$\int_0^1 e^{-x^2}dx = F(b) - F(a) \quad \text{can't do}$$

$$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \dots = \sum_{n=0}^{\infty} \frac{1}{n!}x^n$$

$$e^{-x^2} = \sum_{n=0}^{\infty} \frac{1}{n!}(-x^2)^n = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!}x^{2n}$$

$$(-x^2)^n = \left[(-1)(x^2)\right]^n = (-1)^n x^{2n}$$

$$\int_0^1 e^{-x^2}\,dx = \int_0^1 \left[\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^{2n}\right] dx$$

$\downarrow$ calc

0.7468

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \left[\int_0^1 x^{2n}\,dx\right]$$

0.74319

0.7429

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \left[\frac{1}{2n+1} x^{2n+1}\right]_0^1$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)n!} = 1 - \frac{1}{3} + \frac{1}{5\cdot 2} - \cdots$$

$$= 1 - \frac{1}{3} + \frac{1}{10} - \frac{1}{42} + \cdots$$

Find a series sol for $y''+xy'+2y=0$
using summation notation.

Recall $y=\sum_{n=0}^{\infty} a_n x^n \Rightarrow y'=\sum_{n=1}^{\infty} n a_n x^{n-1}$

$\Rightarrow y''=\sum_{n=2}^{\infty} n(n-1)x^{n-2}$

$$y'' + xy' + 2y = \sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} + x\sum_{n=1}^{\infty} n a_n x^{n-1} + 2\sum_{n=0}^{\infty} a_n x^n$$

$$= \sum_{n=0}^{\infty} (n+2)(n+1)a_{n+2} x^n + \sum_{n=1}^{\infty} n a_n x^n + \sum_{n=0}^{\infty} 2a_n x^n$$

$n=0$ $n=0$

$$= 2a_2 + 2a_0$$

$$+ \sum_{n=1}^{\infty} \left[(n+2)(n+1)a_{n+2} + na_n + 2a_n\right] x^n$$

$2a_2 + 2a_0 = 0$

$a_2 = -a_0$

$$(n+2)(n+1)a_{n+2} + na_n + 2a_n = 0$$

$$\cancel{(n+2)}(n+1)a_{n+2} = -\cancel{(n+2)}a_n$$

$$\boxed{a_{n+2} = -\frac{a_n}{n+1}}$$

$$\boxed{a_{n+2} = -\frac{a_n}{n+1}} \rightsquigarrow \text{recursion formula}$$

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$

$$a_2 = -a_0 \qquad a_3 = -\frac{1}{2}a_1$$

$$a_4 = -\frac{1}{3}a_2 = \frac{1}{3}a_0 \qquad a_5 = -\frac{1}{4}a_3 = \frac{1}{4\cdot 2}a_1$$

$$a_6 = -\frac{1}{5}a_4 = -\frac{1}{5\cdot 3}a_0 \qquad a_7 = -\frac{1}{6\cdot 4\cdot 2}a_1$$

$$y = a_0\left(1 - x^2 + \frac{1}{3}x^4 - \frac{1}{5\cdot 3}x^6 + \cdots\right) + a_1\left(x - \frac{1}{2}x^3 + \frac{1}{4\cdot 2}x^5 - \frac{1}{6\cdot 4\cdot 2}x^7 + \cdots\right)$$