$$\sum \frac{(-1)^{n+1}}{\sqrt{n^2+1}}$$

* What do we <u>suspect</u>?
* What do we <u>know</u>?
* How do we know it?
* <u>When</u> do we know it?

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n^2+1}} \quad \text{very much like} \quad \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$$

Alternating harmonic series

converges

What about $\sum_{n=1}^{\infty} \left| \frac{(-1)^{n+1}}{\sqrt{n^2+1}} \right|$? Like $\sum \frac{1}{n}$ Harmonic, diverges

conditionally convergent

$$\sum \frac{(-1)^{n+1}}{\sqrt{n^2+1}}$$

Alternating series test

$a_{n+1} \le a_n$

$a_n = \frac{1}{\sqrt{n^2+1}}$ must decrease ✓ and have limit 0. ✓

$a_{n+1} = \frac{1}{\sqrt{(n+1)^2+1}}$

$n+1 \ge n$

$(n+1)^2 \ge n^2$

$(n+1)^2+1 \ge n^2+1$

$\sqrt{(n+1)^2+1} \ge \sqrt{n^2+1}$

$\frac{1}{\sqrt{(n+1)^2+1}} \le \frac{1}{\sqrt{n^2+1}}$

$$\lim_{n\to\infty} \frac{1}{\sqrt{n^2+1}} = \lim_{n\to\infty} \frac{1}{\sqrt{n^2}} \qquad 1 << n^2$$

$$= \lim_{n\to\infty} \frac{1}{n} \qquad \sqrt{n^2} = n \text{ if } n \ge 0$$

$$= 0$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n^2+1}}$$ converges, alternating series test

$$\sum \left| \frac{(-1)^{n+1}}{\sqrt{n^2+1}} \right| = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$$ Converges?

Comparison with $\sum \frac{1}{n}$

Direct comparison

$$n^2+1 \geq n^2$$

$$\sqrt{n^2+1} \geq \sqrt{n^2}$$

$$\frac{1}{\sqrt{n^2+1}} \leq \frac{1}{n}$$

Not helpful!

Limit comparison

$$\lim_{n\to\infty} \frac{\frac{1}{\sqrt{n^2+1}}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{\sqrt{n^2+1}}$$

$$= \lim_{n\to\infty} \frac{n}{\sqrt{n^2}}$$

$$= \lim_{n\to\infty} \frac{n}{n} = 1$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n^2+1}}$$ does not converge absolutely.

$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots$$

$f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ converges for all real numbers

$$f'(x) = \sum_{n=0}^{\infty} \left(\frac{x^n}{n!}\right)' = \sum_{n=0}^{\infty} \frac{n x^{n-1}}{n!} = \sum_{n=1}^{\infty} \frac{x^{n-1}}{(n-1)!}$$

$$\frac{\cancel{n}}{\cancel{n}(n-1)(n-2)\cdots}$$

$$\int f(x)\,dx = C + \sum \int \frac{x^n}{n!}\,dx = C + \sum_{n=0}^{\infty} \frac{1}{n+1} \frac{x^{n+1}}{n!}$$

$$= C + \sum_{n=0}^{\infty} \frac{x^{n+1}}{(n+1)!}$$

$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots$$

Geometric, $r = x$, converges $|x| < 1$

radius $= 1$

interval $(-1, 1)$