$$\sum_{n=a}^{\infty} \frac{n}{n^3+1}$$

$$\sum \frac{1}{n^2} \text{ converges, } p=2$$

$$\frac{n}{n^3+1} \overset{<}{>} \frac{1}{n^2} = \frac{n}{n^3}$$

$$n^3+1 > n^3$$

$$\frac{1}{n^3+1} < \frac{1}{n^3}$$

$$\frac{n}{n^3+1} < \frac{n}{n^3} = \frac{1}{n^2}$$

$$\sum_{n=a}^{\infty} \frac{n^4}{n^3+1} \qquad \sum n \quad \text{div}$$

$$\sum_{n=a}^{\infty} \frac{n^3}{n^3+1} \qquad \sum 1 \quad \text{div}$$

$$\sum_{n=a}^{\infty} \frac{n^2}{n^3+1} \qquad \sum \frac{1}{n} \quad \text{dv}$$

$$\sum_{n=a}^{\infty} \frac{n^{\frac{3}{2}}}{n^3+1} \qquad \sum \frac{n^{\frac{3}{2}}}{n^3} = \sum \frac{1}{n^{3/2}} \text{ conv}$$

$$\sum_{n=a}^{\infty} \frac{n^2}{n^3-1} \qquad \sum \frac{1}{n} = \sum \frac{n^2}{n^3}$$

$$\boxed{\begin{gathered} n^3-1 < n^3 \\ \frac{1}{n^3-1} > \frac{1}{n^3} \\ \frac{n^2}{n^3-1} > \frac{n^2}{n^3} \end{gathered}}$$

dir comparison

$$\sum \frac{\sin^2 n}{n^2}$$

compare directly with $\sum \frac{1}{n^2}$

$|\sin x| \leq 1$

$-1 \leq \sin x \leq 1$

$\sin^2 x \leq 1$

limit comparison

$$\boxed{\lim_{n\to\infty} \frac{\frac{\sin^2 n}{n^2}}{\frac{1}{n^2}} = \lim_{n\to\infty} \sin^2 n}$$

doesn't exist

$$\sum_{n=0}^{\infty} \left(\frac{x}{2}\right)^n$$

for convergence

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{\left(\frac{x}{2}\right)^{n+1}}{\left(\frac{x}{2}\right)^n}\right| = \lim_{n\to\infty} \left|\frac{x}{2}\right| = \frac{1}{2}|x| < 1$$

$$|x| < 2 = R$$

$$(-2, 2)$$

$x=2$: $\sum_{n=0}^{\infty} \left(\frac{2}{2}\right)^n = \sum_{n=0}^{\infty} 1$ diverges

$$\sum_{n=0}^{\infty}\left(\frac{x}{2}\right)^n$$

$$\lim_{n\to\infty}\sqrt[n]{\left|\left(\frac{x}{2}\right)^n\right|}=\lim_{n\to\infty}\sqrt[n]{\left|\frac{x}{2}\right|^n}=\lim_{n\to\infty}\left|\frac{x}{2}\right|=\frac{1}{2}|x|<1$$

$$|x|<2$$

Function $f(x)$, could it equal a 4th degree polynomial?

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 \qquad a_0 = f(0)$$

$$f'(x) = a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3 \qquad a_1 = f'(0)$$

$$f''(x) = 2a_2 + 3\cdot 2a_3x + 4\cdot 3a_4x^2 \qquad a_2 = \frac{f''(0)}{2}$$

$$f^{(3)}(x) = 3\cdot 2a_3 + 4\cdot 3\cdot 2a_4x \qquad a_3 = \frac{f'''(0)}{3\cdot 2}$$

$$f(x) = \frac{f^{(0)}(0)}{0!} + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots \qquad a_4 = \frac{f^{(4)}(0)}{4!}$$

Mclaurin polynomial

Find the 3rd deg Mclaurin Polynomial for e^{2x}.

$f(x)=e^{2x} \Longrightarrow f(0)=1$

$f'(x)=2e^{2x} \Longrightarrow f'(0)=2$

$f''(x)=4e^{2x} \Longrightarrow f''(0)=4$

$f'''(x)=8e^{2x} \longrightarrow f'''(0)=8$

$e^{2x} \approx 1+\frac{2}{1!}x+\frac{4}{2!}x^2+\frac{8}{3!}x^3$

$e^{2x} \approx 1+2x+2x^2+\frac{4}{3}x^3$

Turn In:

8.7: 5, 6