$$\int_0^3 e^{x^2}\,dx = \int_0^3 \left[\sum_{n=0}^{\infty} \frac{1}{n!} x^{2n}\right] dx$$

no antiderivative

$$= \sum_{n=0}^{\infty} \frac{1}{n!}\left[\int_0^3 x^{2n}\,dx\right]$$

$$= \sum_{n=0}^{\infty} \frac{3^{2n+1}}{(2n+1)\,n!}$$

$$\int_0^3 x^{2n}\,dx = \left.\frac{x^{2n+1}}{2n+1}\right|_0^3 = \frac{3^{2n+1}}{2n+1}$$

$$\int [f+g]\,dx = \int f\,dx + \int g\,dx$$

$$e^x = \sum_{n=0}^{\infty} \frac{1}{n!} x^n$$

$$e^{x^2} = \sum_{n=0}^{\infty} \frac{1}{n!}\left(x^2\right)^n = \sum_{n=0}^{\infty} \frac{1}{n!} x^{2n}$$

$$\sum_{n=0}^{\infty} \frac{n!}{2^n} x^n$$

$$\lim_{n\to\infty} \frac{\left|\frac{(n+1)!}{2^{n+1}} x^{n+1}\right|}{\left|\frac{n!}{2^n} x^n\right|} = \lim_{n\to\infty} \frac{n+1}{2}|x| = \infty \nless 1$$

unless $x=0$

converges only for $x=0$

$$\sum_{n=0}^{\infty} \frac{2^n}{n!} x^n$$

$$\lim_{n\to\infty} () = \lim_{n\to\infty} \frac{2}{n+1}|x| = 0 < 1$$ no matter what x is.

Converges on $(-\infty, \infty)$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\,3^n}(x-2)^n$$

$\to c=2$

$$\frac{(x-2)^2}{16}+\frac{(y+3)^2}{9}=1$$

$x=2$ is important

$y=-3$ is important

$y'+3y=0 \qquad y=a_0+a_1x+a_2x^2+a_3x^3+\cdots$

$y'=a_1+2a_2x+3a_3x^2+\cdots$

$y'+3y=(a_1+3a_0)+(2a_2+3a_1)x+(3a_3+3a_2)x^2$
$+(4a_4+3a_3)x^3+(5a_5+3a_4)x^4+\cdots$

$a_1+3a_0=0 \qquad 2a_2+3a_1=0 \qquad 3a_3+3a_2=0 \quad \cdots$

$a_1=-3a_0 \qquad 2a_2=-3a_1 \qquad 3a_3=-3a_2$

$a_2=\frac{-3a_1}{2} \qquad a_3=-\frac{3}{3}a_2=\frac{-3^3}{3\cdot 2}a_0$

$=\frac{3^2}{2}a_0$

$y=a_0-3a_0x+\frac{3^2}{2!}a_0x^2-\frac{3^3}{3!}a_0x^3+\cdots=a_0\sum_{n=0}^{\infty}\frac{(-1)^n3^n}{n!}x^n$

$$f(x) = \frac{5}{x-4}$$

Idea:

$$\frac{1}{1-r} = \sum_{n=0}^{\infty} r^n$$

when $|r| < 1$

$$= -5 \cdot \frac{1}{4-x}$$

$$= -\frac{5}{4} \cdot \frac{1}{1-\frac{1}{4}x} \quad r = \frac{1}{4}x$$

$$\left|\frac{1}{4}x\right| < 1$$

$$\frac{1}{4}|x| < 1$$

$$|x| < 4$$

$$= -\frac{5}{4}\sum_{n=0}^{\infty} \frac{1}{4^n} x^n$$

IOC is $(-4, 4)$

$$e^{x} = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots = \sum_{n=0}^{\infty} \frac{1}{n!}x^n \quad (x^2)^n$$

$$e^{-\frac{x^2}{2}} = 1 + \left(-\frac{x^2}{2}\right) + \frac{1}{2!}\left(-\frac{x^2}{2}\right)^2 + \cdots \quad = \sum_{n=0}^{\infty} \frac{1}{n!}\left(-\frac{x^2}{2}\right)^n$$

$$= 1 - \frac{1}{2}x^2 + \frac{1}{2!\,2^2}x^4 \quad = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n n!} x^{2n}$$

Hyperbolic (trig) functions

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2}$$

$$= \frac{1}{2}(e^x - e^{-x})$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

$$(\sin x)' = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots = \cos x$$

$\cosh^2 x$ means $(\cosh x)^2$