

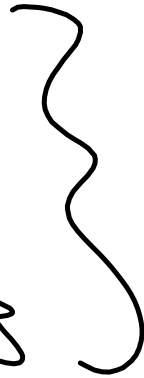
Final Wed 10-12 here

15% Exam 1

35% Exam 2

35% Exam 3

$\Rightarrow$ 15% since Exam 3



$$\begin{aligned}
 & x \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} + \sum_{n=0}^{\infty} a_n x^n \\
 & \stackrel{m=n-1}{\stackrel{n=m+1}{\sum_{n=2}^{\infty}}} n(n-1) a_n x^{n-1} + \sum_{n=0}^{\infty} a_n x^n \\
 & \sum_{m=1}^{\infty} (m+1)m a_{m+1} x^m + \sum_{n=0}^{\infty} a_n x^n
 \end{aligned}
 \quad
 \begin{aligned}
 y &= \sum_{n=0}^{\infty} a_n x^n \\
 y' &= \sum_{n=1}^{\infty} n a_n x^{n-1} \\
 y'' &= \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}
 \end{aligned}$$

$$a_0 + \sum_{n=1}^{\infty} [(n+1)n a_{n+1} + a_n] x^n = 0 \quad \text{if from } xy'' + y = 0$$

$$\boxed{(n+1)n a_{n+1} + a_n = 0} \quad \text{recursion formula}$$

$= 0$ if from $xy'' + y = 0$

$$a_{n+1} = -\frac{1}{(n+1)n} a_n$$

$$a_1 = a_1$$

$$a_2 = -\frac{1}{2} a_1$$

$$a_3 = -\frac{1}{3 \cdot 2} a_2 = \frac{1}{3 \cdot 2 \cdot 2} a_1$$

$$a_4 = -\frac{1}{4 \cdot 3} a_3 = -\frac{1}{4 \cdot 3 \cdot 3 \cdot 2 \cdot 2} a_1$$

$$y'' + y = 0$$

$$y'' - y = 0$$

$$\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} \pm \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=0}^{\infty} (n+2)(n+1)a_{n+2} x^n \pm \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=0}^{\infty} [(n+2)(n+1)a_{n+2} \pm a_n] x^n = 0$$

$$y = a_0 + a_1 x + a_2 x^2 + \dots \quad (n+2)(n+1)a_{n+2} \pm a_n = 0$$

$$a_{n+2} = \mp \frac{1}{(n+2)(n+1)} a_n$$

$$a_0 = a_0$$

$$a_1 = a_1$$

$$a_2 = -\frac{1}{2 \cdot 1} a_0 = -\frac{1}{2!} a_0$$

$$a_3 = -\frac{1}{3 \cdot 2} a_1$$

$$a_4 = -\frac{1}{4 \cdot 3} a_2 = \frac{1}{4!} a_0$$

$$a_5 = \frac{1}{5!} a_1$$

$$a_6 = -\frac{1}{6!} a_0$$

$$a_7 = -\frac{1}{7!} a_1$$

$$y = a_0 \left(1 - \frac{1}{2!} x^2 + \frac{1}{4!} x^4 - \dots \right) + a_1 \left(x - \frac{1}{3!} x^3 + \frac{1}{5!} x^5 - \dots \right)$$

$\underbrace{\hspace{10em}}_{\cos x} \qquad \underbrace{\hspace{10em}}_{\sin x}$

Solution to $y'' - y = 0$

$$y = a_0 \left(1 + \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \dots \right) + a_1 \left(x + \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \dots \right)$$
$$\cosh x = \frac{e^x + e^{-x}}{2} \quad \sinh x = \frac{e^x - e^{-x}}{2}$$

Trig sin & cos

$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 -$$

$$\cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 -$$

$$(\sin x)' = \cos x, (\cos x)' = -\sin x$$

$$\text{Sols to } y'' + y = 0$$

$$\sin^2 x + \cos^2 x = 1$$

Hyperbolic sinh & cosh

$$\sinh x = x + \frac{1}{3!}x^3 + \frac{1}{5!}x^5 + \dots$$

$$\cosh x = 1 + \frac{1}{2!}x^2 + \frac{1}{4!}x^4 + \dots$$

$$(\sinh x)' = \cosh x$$

$$(\cosh x)' = \sinh x$$

$$\text{Sols to } y'' - y = 0$$

$$\cosh^2 x - \sinh^2 x = 1$$

