Final Wed 10-12 here

15% Exam 1
35% Exam 2
35% Exam 3
⟹ 15% since Exam 3

$$x\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} + \sum_{n=0}^{\infty} a_n x^n \qquad y=\sum_{n=0}^{\infty} a_n x^n$$

$$y'=\sum_{n=1}^{\infty} n a_n x^{n-1}$$

$m=n-1$
$n=m+1$

$$\sum_{n=2}^{\infty} n(n-1)a_n x^{n-1} + \sum_{n=0}^{\infty} a_n x^n$$

$$y''=\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2}$$

$$\sum_{m=1}^{\infty} (m+1)m a_{m+1} x^m + \sum_{n=0}^{\infty} a_n x^n$$

$$a_0 + \sum_{n=1}^{\infty}\Big[\underbrace{(n+1)n a_{n+1} + a_n}\Big]x^n = 0 \text{ if from } xy''+y=0$$

$=0$ if from $xy''+y=0$

$$\boxed{(n+1)n a_{n+1} + a_n = 0}$$ recursion formula

$$a_{n+1} = -\frac{1}{(n+1)n} a_n$$

$$a_1 = a_1$$

$$a_2 = -\frac{1}{2}a_1$$

$$a_3 = -\frac{1}{3\cdot 2}a_2 = \frac{1}{3\cdot 2\cdot 2}a_1$$

$$a_4 = -\frac{1}{4\cdot 3}a_3 = -\frac{1}{4\cdot 3\cdot 3\cdot 2\cdot 2}a_1$$

$y'' + y = 0 \qquad\qquad y'' - y = 0$

$m = n-2$
$n = m+2$

$$\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} \pm \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=0}^{\infty} (n+2)(n+1)a_{n+2} x^n \pm \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=0}^{\infty} \left[(n+2)(n+1)a_{n+2} \pm a_n\right] x^n = 0$$

$$(n+2)(n+1)a_{n+2} \pm a_n = 0$$

$y = a_0 + a_1 x + a_2 x^2 + \cdots$

$$a_{n+2} = \mp \frac{1}{(n+2)(n+1)} a_n$$

$a_0 = a_0 \qquad a_1 = a_1$

$a_2 = -\frac{1}{2\cdot 1} a_0 = -\frac{1}{2!} a_0 \qquad a_3 = -\frac{1}{3!} a_1$

$a_4 = -\frac{1}{4\cdot 3} a_2 = \frac{1}{4!} a_0 \qquad a_5 = \frac{1}{5!} a_1$

$a_6 = -\frac{1}{6!} a_0 \qquad a_7 = -\frac{1}{7!} a_1$

$$\boxed{y = a_0 \underbrace{\left(1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \cdots\right)}_{\cos x} + a_1 \underbrace{\left(x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \cdots\right)}_{\sin x}}$$

Solution to $y'' - y = 0$

$$y = a_0\underbrace{\left(1 + \frac{1}{2!}x^2 + \frac{1}{4!}x^4 + \cdots\right)}_{\cosh x = \frac{e^x + e^{-x}}{2}} + a_1\underbrace{\left(x + \frac{1}{3!}x^3 + \frac{1}{5!}x^5 + \cdots\right)}_{\sinh x = \frac{e^x - e^{-x}}{2}}$$

Trig sin + cos

$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 -$

$\cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 -$

$(\sin x)' = \cos x$, $(\cos x)' = -\sin x$

Sols to $y'' + y = 0$

$\sin^2 x + \cos^2 x = 1$

Hyperbolic sinh + cosh

$\sinh x = x + \frac{1}{3!}x^3 + \frac{1}{5!}x^5 + \cdots$

$\cosh x = 1 + \frac{1}{2!}x^2 + \frac{1}{4!}x^2 + \cdots$

$(\sinh x)' = \cosh x$

$(\cosh x)' = \sinh x$

Sols to $y'' - y = 0$

$\cosh^2 x - \sinh^2 x = 1$