1) Find a Taylor series centered at $c=2$ for $f(x) = \dfrac{3}{x+1}$. Find the IOC.

2) (Harder?) Find a Taylor series centered at $c=-1$ for $g(x) = \dfrac{2}{3x-5}$

bad $3x-5=0$

$3x=5$

$x=\frac{5}{3}$

-1 $\quad$ $\frac{5}{3}$

$\left(-\frac{11}{3}, \frac{5}{3}\right)$ is IOC

$$g(x) = \frac{2}{3x-5}$$

$$= -2 \cdot \frac{1}{5-3x}$$

$$y = 3x^2 - 12x + 2$$

$$= -2 \cdot \frac{1}{5-3(x+1-1)}$$

$$y = 3(x^2-4x)+2$$

$$= -2 \cdot \frac{1}{5-3(x+1)+3}$$

$$y = 3(x^2-4x+4-4)+2$$

$$= 3(x^2-4x+4)-12+2$$

$$= -2 \frac{1}{8-3(x+1)}$$

$$= -\frac{2}{8} \cdot \frac{1}{1-\frac{3}{8}(x+1)}$$

$$\left|\frac{3}{8}(x+1)\right| < 1$$

$$= -\frac{1}{4} \frac{1}{1-\frac{3}{8}(x+1)}$$

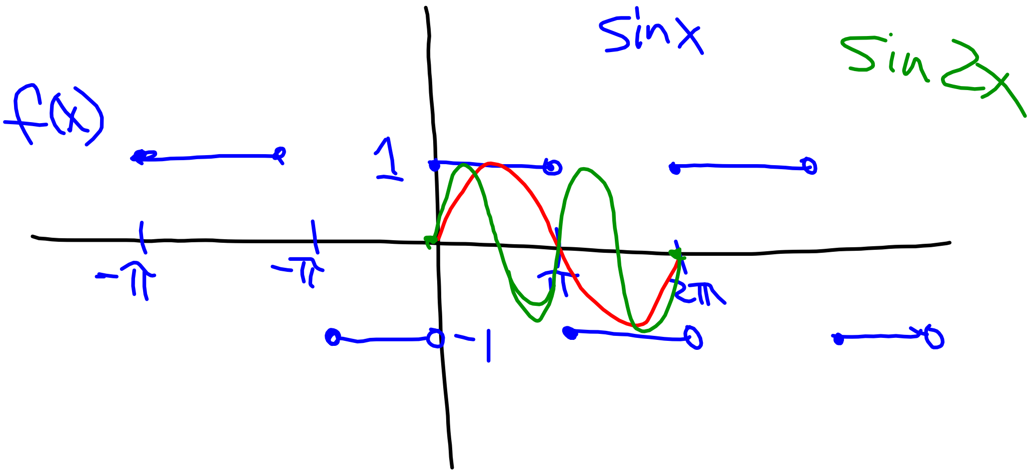
$$r = \frac{3}{8}(x+1)$$

$$|x+1| < \frac{8}{3}$$

$$= -\frac{1}{4} \sum_{n=0}^{\infty} \left(\frac{3}{8}\right)^n (x+1)^n$$

$$|x-(-1)| < \frac{8}{3}$$

Fourier Series



$$f(x) = a_1 \sin x + a_2 \sin 2x + a_3 \sin 3x + \cdots$$

$$\int_0^{2\pi} \sin mx \sin nx\,dx = \begin{cases} 0 & \text{if } m \neq n \\ 2\pi & \text{if } m = n \end{cases}$$

$$f(x)\sin 2x = a_1 \sin x \sin 2x + a_2 \sin 2x \sin 2x + a_3 \sin 3x \sin 2x + \cdots$$

$$\int_0^{2\pi} f(x)\sin 2x\,dx = a_1 \cancelto{0}{\int_0^{2\pi} \sin x \sin 2x\,dx} + a_2 \int_0^{2\pi} \sin 2x \sin 2x\,dx + a_3 \cancelto{0}{\int_0^{2\pi} \sin 3x \sin 2x\,dx}$$

$$\int_0^{2\pi} f(x)\sin 2x\,dx = 2\pi a_2$$

$$\underset{n}{a_{\cancel{x}}} = \frac{1}{2\pi}\int_0^{2\pi} f(x)\sin \underset{n}{\cancel{2}}x\,dx$$

$$\int_0^{2\pi} x \sin nx\,dx \qquad y = mx + \cancel{b}$$

$$\int_0^{2\pi} x \sin nx \, dx$$

Let $u = x$ $\quad dv = \sin nx \, dx$

du $\quad v$

$$\int_a^b u \, dv = uv\Big|_a^b - \int_a^b v \, du$$

$$\int \sin nx \, dx = -\frac{1}{n} \cos nx$$

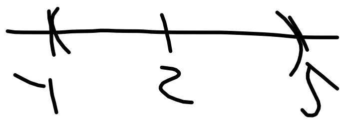
mathispower4u

① $f(x) = \dfrac{3}{x+1} = \dfrac{b}{c} \cdot \dfrac{1}{1-a(x-2)}$

$f(x) = \dfrac{3}{x+1} = \dfrac{3}{1+x+2-2}$

bad $x=-1$

$= \dfrac{3}{3+x-2}$ $\qquad \dfrac{3}{1+x-2}$



$= \dfrac{3}{3-[-(x-2)]}$ $\qquad \dfrac{3}{3+x-2}$

Converges for

$|r|<1$

$\left|-\frac{1}{3}(x-2)\right|<1$

$\frac{1}{3}|x-2|<3$

$|x-2|<3$

$= \cancel{\dfrac{3}{3}} \cdot \dfrac{1}{1-\left[-\frac{1}{3}(x-2)\right]}$ $\qquad \dfrac{1}{1-r}$

$r = -\frac{1}{3}(x-2)$

$= \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{3^n}(x-2)^n$

IOC is $(-1,5)$

$$f(x) = \frac{3}{x+1}$$

$f(x) = 3(x+1)^{-1}$ $\quad f(2) = \frac{3}{3} = 1$

$f'(x) = -3(x+1)^{-2}$ $\quad f'(2) = -\frac{3}{9} = -\frac{1}{3}$

$f''(x) = 6(x+1)^{-3}$ $\quad f''(2) = \frac{6}{27} = \frac{2}{9}$ $\qquad \frac{1}{20}$

$f'''(x) = -18(x+1)^{-4}$ $\quad f''' = -\frac{\cancel{3}\cdot 3\cdot 2}{3^{\cancel{4}}} = -\frac{3!}{3^3}$