Math 253N ASSIGNMENT 14 **Due at the start of class Monday, May 15th**

Do all work on additional paper.

1. On a previous assignment it was determined that the seventh degree Mclaurin polynomial (Taylor polynomial centered at zero) is

$$p_7(x) = x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7.$$

   (a) Use the polynomial to approximate $\sin 2$. **Give your result to four places past the decimal by writing** $\sin 2 \approx answer$.

   (b) Find the error bound $\frac{M}{(n+1)!}|x-c|^{n+1}$, where M is the maximum of $|f^{(n+1)}(x)|$ on the interval from c to x. Show clearly the values used for each part of this computation. **Round to four places past the decimal**.

   (c) Find the "actual" value of $\sin 2$ using your calculator. (Be sure your calculator is in radians!) Give both the actual value and the error.

   (d) If the actual error is not less than your error bound, go back and find the problem and fix it if you can.

2. The point of this exercise is to see that if we center our Taylor series closer to where we want our approximation, the result is a smaller error for a Taylor polynomial of the same degree, or a similar or still smaller error for a Taylor polynomial of lesser degree.

   (a) Carefully find the Taylor polynomial of degree four, centered at $\frac{\pi}{2}$, for the sine function. Show all derivatives and their values at $\frac{\pi}{2}$, as we've been doing.

   (b) Use the Taylor polynomial you found to approximate $\sin 2$.

   (c) Find the error bound, this time to six places past the decimal.

   (d) Give the actual error for the approximation from part (b). Again, if it is not less than the error bound, find and correct any mistakes.

3. The third degree Mclaurin polynomial for e^x is $p_3(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3$.

   (a) Use this to approximate $e^{-0.5}$.

   (b) Give the error bound, again showing how you obtain it.

   (c) Give the actual error.