Math 253N ASSIGNMENT 16 **Due at the start of class Monday, May 22nd**

Do all work on additional paper.

1. Find a series solution for the differential equation $y' + 2xy = 0$. Give your final answer in summation form.

2. **Algebraically** determine the Taylor series centered at $c = 2$ for $h(x) = \dfrac{3}{7+x}$ and the interval in which the series converges to the function. Give your final answer for the series in summation form.

3. In this exercise you will work with the Mclaurin series for the sine and cosine functions.

   (a) Write out the first five terms of the series for sine, and the one for cosine.

   (b) What do $(-x)^3$ and $(-x)^4$ equal?

   (c) Write out the first five terms for $\cos(-x)$, then simplify using something like your result from (b). How does your result compare with the series for cosine? Summarize with a concise statement.

   (d) Repeat (c) for the sine function.

4. (a) Remember sequences? They are just infinite lists of numbers. Write out the first ten terms of the sequence $\left\{(i)^n\right\}_{n=0}^{\infty}$, where $i^2 = -1$. Separate the terms of the sequence with commas.

   (b) Write out the first ten terms of the Mclaurin series for $e^{i\theta}$. Simplify each term using your results from part (a).

   (c) Some of the terms of your series should have i in them and some shouldn't. Rearrange the terms of the series to get the terms without i together, and the ones with i together. Conclude each portion with $\cdots$.

   (d) Factor i out of the terms that have i in them. You should now have the sum of two separate series, one of them multiplied by i, and you should recognize those series.

   (e) What is $e^{i\theta}$ equal to, in terms of familiar functions?