① $y' + 2xy = 0$

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + \cdots$$
$$y' = a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + 5a_5 x^4 + 6a_6 x^5 + \cdots$$

$$y' + 2xy = a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + 5a_5 x^4 + 6a_6 x^5 + \cdots$$
$$+ 2a_0 x + 2a_1 x^2 + 2a_2 x^3 + 2a_3 x^4 + 2a_4 x^5 + \cdots$$
$$= a_1 + (2a_2 + 2a_0)x + (3a_3 + 2a_1)x^2 + (4a_4 + 2a_2)x^3 + \cdots$$

$a_1 = 0$ $\quad a_2 = -\frac{2}{2}a_0 = -a_0$ $\quad a_3 = 0$ $\quad a_4 = -\frac{2}{4}a_2 = \frac{2^2}{4 \cdot 2}a_0 = \frac{1}{2}a_0$ $\quad a_5 = 0$

$6a_6 = -2a_4$

$a_6 = -\frac{2}{6}a_4 = -\frac{1}{3!}a_0$

$$\boxed{y = a_0\left(1 - x^2 + \frac{1}{2}x^4 - \frac{1}{3!}x^6 + \cdots\right) \qquad y = a_0 \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^{2n}}$$

②

$$h(x) = \frac{3}{7+x} = \frac{3}{7+x+2-2} = \frac{3}{9+(x-2)} = \frac{1}{3} \cdot \frac{1}{1+\frac{1}{9}(x-2)} = \frac{1}{3} \cdot \frac{1}{1-[-\frac{1}{9}(x-2)]}$$

Geometric, $r = -\frac{1}{9}(x-2)$, so $\boxed{h(x) = \frac{1}{3}\sum_{n=0}^{\infty}\left(-\frac{1}{9}\right)^n (x-2)^n}$

Converges for $\left|-\frac{1}{9}(x-2)\right| < 1$, or $|x-2| = 9$. Centered at $c=2$, ROC $= 9$. The IOC is $\boxed{(-7, 11)}$

③ a) $\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \frac{1}{9!}x^9 - \cdots$

$\cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \frac{1}{8!}x^8 \cdots$

b) $(-x)^3 = -x$, $(-x)^4 = x$

(3) c) $\cos(-x) = 1 - \frac{1}{2!}(-x)^2 + \frac{1}{4!}(-x)^4 - \frac{1}{6!}(-x)^6 + \frac{1}{8!}(-x)^8 - \cdots$

$= 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \frac{1}{8!}x^8 - \cdots$

The series is the same as the one for $\cos x$

d) $\sin(-x) = (-x) - \frac{1}{3!}(-x)^3 + \frac{1}{5!}(-x)^5 - \frac{1}{7!}(-x)^7 + \frac{1}{9!}(-x)^9 - \cdots$

$= -x + \frac{1}{3!}x^3 - \frac{1}{5!}x^5 + \frac{1}{7!}x^7 - \frac{1}{9!}x^9 + \cdots$

The series for $\sin(-x)$ is the negative of the series for $\sin x$.

(4)
a) $\{(i)^n\}_{n=0}^{\infty} = 1, i, -1, -i, 1, i, -1, -i, 1, i, \ldots$

b) $e^{i\theta} = 1 + (i\theta) + \frac{1}{2!}(i\theta)^2 + \frac{1}{3!}(i\theta)^3 + \frac{1}{4!}(i\theta)^4 +$

$+ \frac{1}{5!}(i\theta)^5 + \frac{1}{6!}(i\theta)^6 + \frac{1}{7!}(i\theta)^7 + \frac{1}{8!}(i\theta)^8 + \frac{1}{9!}(i\theta)^9 + \cdots$

$= 1 + i\theta - \frac{1}{2!}\theta^2 - i\frac{1}{3!}\theta^3 + \frac{1}{4!}\theta^4 + i\frac{1}{5!}\theta^5$

$- \frac{1}{6!}\theta^6 - i\frac{1}{7!}\theta^7 + \frac{1}{8!}\theta^8 + i\frac{1}{9!}\theta^9 + \cdots$

$= \left(1 - \frac{1}{2!}\theta^2 + \frac{1}{4!}\theta^4 - \frac{1}{6!}\theta^6 + \frac{1}{8!}\theta^8 - \cdots\right)$

$+ i\left(\theta - \frac{1}{3!}\theta^3 + \frac{1}{5!}\theta^5 - \frac{1}{7!}\theta^7 + \frac{1}{9!}\theta^9 - \cdots\right)$

$$\boxed{e^{i\theta} = \cos\theta + i\sin\theta}$$