Math 253N Assignment 2 **Due at the start of class on Monday, April 10th**

In this assignment you will be asked for a number of limits. **In each case, give your answer by writing the limit asked for, followed by all steps used in obtaining the value of the limit**. If the limit does not exist, write the limit asked for followed by DNE, for does not exist. In the cases where the a limit is infinity or negative infinity, you may write the limits as ∞ or $-\infty$, but you need to remember that *technically speaking these limits do not exist.* **All work and answers should be shown on an additional sheet of paper**.

1. (a) Find $\lim\limits_{x\to\infty} \dfrac{x^2-5x}{3x^2+2}$ by using algebra to replace the given limit with an equivalent one.

   (b) Find $\lim\limits_{x\to\infty} \dfrac{x^2-5x}{3x^2+2}$ by using L'Hôpital's rule..

2. Find $\lim\limits_{R\to\infty} \left(1-\dfrac{1}{R}\right)$.

3. Find $\lim\limits_{x\to\infty} x\,e^{-x}$ by first rewriting without a negative exponent, then doing something "formal."

4. For this exercise we will consider the function $h(x) = \dfrac{\sin(2x)}{x}$.

   (a) This function can be thought of as $h(x) = \dfrac{f(x)}{g(x)}$. What are $\lim\limits_{x\to 0} f(x)$ and $\lim\limits_{x\to 0} g(x)$? There are no steps that need to be shown for these.

   (b) Why can we not apply $\lim\limits_{x\to 0} h(x) = \dfrac{\lim\limits_{x\to 0} f(x)}{\lim\limits_{x\to 0} g(x)}$ to find $\lim\limits_{x\to 0} h(x)$?

   (c) Apply L'Hopital's Rule to find $\lim\limits_{x\to 0} h(x)$. (Don't forget about the chain rule!)

   (d) Sketch the graphs of $y = \sin(2x)$ and $y = x$ on the same grid, for $-0.1 \leq x \leq 0.1$. You may use your calculator or *Desmos* to get the graph. Your graph does not need to be beautiful, but each axis should be labeled with a couple numerical values and the graph should indicate what is going on here pretty clearly. you should also indicate which graph is which function.

There are more on the other side!

5. Again consider the function $h(x) = \dfrac{\sin(2x)}{x}$.

   (a) Does $\lim_{x\to\infty} \sin(2x)$ exist? If it does, give its value, showing how you got it. If it doesn't, explain why.

   (b) Does $\lim_{x\to\infty} x$ exist? If it does, give its value, showing how you got it. If it doesn't, explain why.

   (c) Give two functions $p(x)$ and $q(x)$ such that $p(x) \le \dfrac{\sin(2x)}{x} \le q(x)$ for $x \ge 1$.
   Hint: What do you know about the range of $\sin(2x)$?

   (d) Sketch graphs of $h(x)$, $p(x)$ and $q(x)$ all on the same grid, for $x \ge 1$. (**Hint:** Graph $p(x)$ and $q(x)$ first.) Follow the same expectations as given for Exercise 4(d).

   If your graph does not illustrate that $p(x) \le \dfrac{\sin(2x)}{x} \le q(x)$ for $x \ge 1$, you should change your choices of $p(x)$ and/or $q(x)$!

   (e) What is $\lim_{x\to\infty} p(x)$? What is $\lim_{x\to\infty} q(x)$?

   (f) What is $\lim_{x\to\infty} h(x)$? Which theorem does this illustrate?

6. Find each of the following integrals "by hand," showing all of your work.

   (a) $\displaystyle\int_R^2 \frac{1}{x^3}\,dx$ (b) $\displaystyle\int_1^R \frac{1}{x^3}\,dx$

   (c) $\displaystyle\int_0^R e^{-2x}\,dx$ (d) $\displaystyle\int_1^R \frac{1}{\sqrt{x}}\,dx$

7. Determine whether each of the following improper integrals converges or diverges, using the definition of an improper integral and your answers to Exercise 6.

   (a) $\displaystyle\int_0^2 \frac{1}{x^3}\,dx$ (b) $\displaystyle\int_1^\infty \frac{1}{x^3}\,dx$

   (c) $\displaystyle\int_0^\infty e^{-2x}\,dx$ (d) $\displaystyle\int_1^\infty \frac{1}{\sqrt{x}}\,dx$