Math 253N Assignment 3, Spring 2017 **Due at the start of class on April 10th**

1. Compute $\int_4^\infty e^{-3x}\,dx$ as follows, showing your work clearly **on extra paper**:
   - Find the integral from 4 to some value R.
   - Apply the definition of an integral to infinity to your answer to (a). **Give your answer in exact form - no decimals!**

You will need the sequences below for some of the exercises that follow. Refer to the definitions in your packet, and **interpret them exactly as they are written**.

(a) $1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \frac{1}{16}, \ldots$ (b) $3, 3, 3, 3, 3, \ldots$ (c) $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \ldots$

(d) $-\frac{1}{2}, -\frac{1}{4}, -\frac{1}{6}, -\frac{1}{8}, \ldots$ (e) $1, 3, 5, 7, \ldots$ (f) $1, -1, 1, -1, 1, -1, \ldots$

(g) $\frac{1}{2}, \frac{1}{6}, \frac{1}{12}, \frac{1}{20}, \frac{1}{30}, \ldots$ (h) $1, -2, 3, -4, 5, -6, 7, -8, \ldots$ (i) $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \ldots$

2. Give the letters of all of the sequences that are increasing:

3. Give the letters of all of the sequences that are nondecreasing:

4. Give the letters of all of the sequences that are monotonic:

5. Give the letter of each sequence that is bounded, followed by the smallest possible value of a bound M.

6. Give the letters of all of the sequences that have limits, and give the limit of each, letting L represent the limit of a sequence.

You will continue with the sequences from the other side:

(a) $1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \frac{1}{16}, \ldots$ (b) $3, 3, 3, 3, 3, \ldots$ (c) $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \ldots$

(d) $-\frac{1}{2}, -\frac{1}{4}, -\frac{1}{6}, -\frac{1}{8}, \ldots$ (e) $1, 3, 5, 7, \ldots$ (f) $1, -1, 1, -1, 1, -1, \ldots$

(g) $\frac{1}{2}, \frac{1}{6}, \frac{1}{12}, \frac{1}{20}, \frac{1}{30}, \ldots$ (h) $1, -2, 3, -4, 5, -6, 7, -8, \ldots$ (i) $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \ldots$

7. Give the explicit formula for each sequence, including the values of n for which it holds. Remember that alternating signs are obtained by powers of -1, even numbers are obtained by $2n$ and odd numbers by $2n+1$ or $2n-1$.

(a) (b)

(c) (d)

(e) (f)

(g) (h)

(i)

8. Apply Theorem 8.1 and an algebraic method or L'Hopital's rule to find the limit of each of the following sequences, **showing work on additional paper.** Check your answers using *Excel* or the *Desmos Sequence Plotter*.

(a) $\left\{\dfrac{n^2}{n^3+n}\right\}$ (b) $\left\{\dfrac{5n^2+1}{3n^2-1}\right\}$ (c) $\left\{\dfrac{3e^n-1}{3e^n}\right\}$

9. Use *Excel* or the *Desmos Sequence Plotter* that you can find at the web page to determine the limit of the sequence $\left\{n \sin \dfrac{1}{n}\right\}$. $L =$

10. Find the limit of $\left\{\dfrac{5^{n+1}+5}{5^n}\right\}$ without using L'Hopital's rule, showing on extra paper how you do it.

11. Find the limit of $\left\{\dfrac{3e^{2n}-1}{3e^n}\right\}$, showing on extra paper how you do it.