Math 253N DAY 2 ACTIVITY **Spring 2017**

In this activity you will use *Excel* and an online tool to investigate sequences and series.

1. For this exercise we will consider the sequence defined recursively and explicitly by

$$a_{n+1} = \frac{1}{2}a_n\,; \quad a_0 = 1 \qquad \text{and} \qquad a_n = \left(\frac{1}{2}\right)^n\,; \quad n = 0, 1, 2, 3, \ldots$$

   (a) Find the first few terms of the sequence by hand, as decimals, to make sure you know what you are supposed to be getting.

   (b) Using *Excel*, put n in cell A1 and a_n or a_n in cell B1.

   (c) Enter zero and one into cells A2 and A3. If you highlight those two cells you can drag them down by the lower right corner to get all the indices up to some value like, say, twenty. Do that.

   (d) Now you will generate the sequence recursively. Enter the a_0 value in cell B2. Then enter the recursion formula in cell B3, referencing cell B2. *Your formula must start with an = sign.*

   (e) If you then drag that cell down, you should be able to generate the entire sequence. (If it doesn't match what you got in part (a) you should of course find your error!) *Note that the values in column A were not really needed to create the sequence this way.*

2. Now you will generate the same sequence explicitly.

   (a) Enter the column headers n and a_n in cells D1 and E1, and generate the indices in column D in the same way that you did in column A.

   (b) In cell E2, enter the explicit formula that references cell D2 to obtain the first term of the sequence. Copy that cell downward to generate the zeroth through twentieth terms of the sequence.

3. Consider now the series $\displaystyle\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n$ whose terms are the sequence from the previous exercises. We will now generate the sequence of partial sums for this series, storing them in column F. Remembering that the series is

$$a_0 + a_1 + a_2 + a_3 + a_4 + \cdots,$$

   we see that the partial sums can be written as

$$S_0 = a_0, \qquad S_1 = a_0 + a_1 = S_0 + a_1, \qquad S_2 = a_0 + a_1 + a_2 = S_1 + a_2, \qquad S_n = S_{n-1} + a_n.$$

   (a) Add a header of S_n or S_n in cell F1.

   (b) Put the value of S_0 in cell F2. In F3, put a formula that calculates S_1 using S_0 and a_1, as shown above.

   (c) You should now be able to copy your formula in cell F3 downward to obtain the sequence of partial sums.

   (d) What does the limit of partial sums seem to be?

4. You will now use an online tool to work with the series $\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n$.

   (a) For the tool we are going to use, we need to rewrite the series so that the index runs from one to infinity, rather than zero to infinity. Rewrite the summation form of the series and make sure that your result gives the same series.

   (b) Go to the class web page and click the link to the Desmos Sequence/Series Grapher. Pull the slider in line 3 back to $n = 5$.

   (c) Enter the formula for your a_ns from part (a) in line 5, where it asks for a_N. (You will enter it in place of arctan N.)

   (d) Observe carefully what you see. The purple dots are the terms a_n and the red dots are the partial sums S_n. $a_0 = 1$ is covered up by the red dot for $S_1 = a_1 = 1$. You should move the n slider to see fewer and more terms. What do the terms of the sequence a_1, a_2, a_3, ... converge to? What does the sequence of partial sums converge to? Both should agree with what you saw when you were using *Excel*.

5. You will now investigate the series $\sum_{n=1}^{\infty} 5\left(\frac{2}{3}\right)^{n-1}$ from an exercise on Assignment 1.

   (a) Use *Excel* to generate the sequence of terms of the series, and the sequence of partial sums. What does that lead you to believe the *series* converges to?

   (b) Investigate the same series using the Desmos Sequence/Series Plotter. Does it verify what you determined in part (a)?