Math 254 ASSIGNMENT 11, SPRING 2018 **Due at 12:00 on 5/14**

The word "extrema" is the plural of both minimum and maximum. When you are asked to find and describe all local extrema, it means find all maximums or minimums, tell which they are, where they occur and what the function value there is.

1. In this exercise you will find and describe local extrema of the function

$$f(x,y) = 5 + 4x - 2x^2 + 4y - y^2.$$

   (a) Find $f_x(x,y)$ and $f_y(x,y)$ and set each equal to zero and solve. The values obtained are the coordinates of the single critical point we have in this case. Give the critical point (x_0, y_0) by writing *critical point:*, followed by what it is.

   (b) Find the three second partial derivatives $f_{xx}(x,y)$, $f_{yy}(x,y)$ *and* $f_{xy}(x,y)$. Evaluate each at the critical point (x_0, y_0) and compute $D = f_{xx}(x_0,y_0)f_{yy}(x_0,y_0) - f_{xy}^2(x_0,y_0)$. What does the second derivative test (found on the formula sheet) tell us is happening at the critical point?

   (c) If the critical point is a maximum or minimum, calculate the function value there. Then **conclude with a sentence** telling whether the function has a maximum or minimum (or neither), what value it is, and where it occurs. (For an example of such a sentence, see the class notes from 5/11.)

2. Consider the function $f(x,y) = 4x - 3x^3 - 2xy^2$, which has derivatives

$$f_x(x,y) = 4 - 9x^2 - 2y^2, \qquad f_y(x,y) = -4xy,$$

$$f_{xx}(x,y) = -18x, \qquad f_{yy}(x,y) = -4x, \qquad f_{xy}(x,y) = -4y$$

   (a) Make sure you can see how those partial derivatives are obtained.

   (b) By some slightly tricky algebra we can find that the critical points are

$$(0,\sqrt{2}), \qquad (0,-\sqrt{2}), \qquad (\tfrac{2}{3},0), \qquad (-\tfrac{2}{3},0)$$

   If you are up for a bit of a challenge, see if you can figure out how to obtain these. If you don't want to do that, just move on to part (c).

   (c) Apply the second partials test to determine what is happening at each critical point. Write a sentence for each.

3. In this exercise you will find and describe local extrema of the function

$$f(x,y) = xy - x^2 - y^2 - 2x - 2y + 4.$$

   (a) Find the critical points, by solving the system of two equations in two unknowns that you get when you set both $f_x(x,y) = 0$ and $f_y(x,y) = 0$. Use the addition method or substitution, **showing your work**. You should get the point $(-2,-2)$.

   (b) Apply the second partials test and conclude with a complete sentence $f(x,y)$ that summarizes all that you have found out.