1. In this exercise you will find and describe local extrema of the function

$$f(x,y) = \tfrac{1}{3}x^3 + y^2 + 2xy - 6x - 3y + 4.$$

   (a) Find $f_x(x,y)$ and $f_y(x,y)$ and set each equal to zero. This gives us a *nonlinear* system of two equations, meaning at least one of the two equations is not the equation of a line. You can solve it one of two ways: (1) subtract the $f_y(x,y) = 0$ equation from the $f_x(x,y) = 0$ equation and solve for x, or (2) solve the $f_y(x,y) = 0$ equation for y and substitute into the $f_x(x,y) = 0$ equation to solve for x. Do one or the other. Once you have found your x values, substitute them into either $f_y(x,y) = 0$ or $f_x(x,y) = 0$ to obtain y. Give the resulting critical points.

   (b) Find the three second partial derivatives $f_{xx}(x,y)$, $f_{yy}(x,y)$ *and* $f_{xy}(x,y)$, and multiply them as they are to obtain $D = f_{xx}(x,y)f_{yy}(x,y) - f_{xy}^2(x,y)$ for any general point (x,y). Determine the value of D at each critical point, and use the rest of the second derivative test to give an overall conclusion (**sentence!**). Be sure to give any minimum or maximum values of the function, as well as their locations.

2. In this exercise you will find the absolute maxima and minima of $f(x,y) = x^2 + y^2 - 4x - 6y + 13$ on the region $0 \leq x \leq 3,\ 0 \leq y \leq 4$. **This is very much like the example from class.**

   (a) Sketch and shade the region on a set of xy-axes.

   (b) Determine the locations of any critical points in the interior of the region.

   (c) Set $x = 0$ and set the derivative of $f(0,y)$ equal to zero and solve to find any critical points along the edge where $x = 0$.

   (d) Repeat (c) for the edges where $x = 3$, $y = 0$, and $y = 4$.

   (e) Evaluate the function at each of your interior and edge critical points, along with the corners of the region. Organize your work in some way that is easily read.

   (f) Write a concluding statement giving all maxima and minima of the function and where they occur.

3. In this exercise you will find the absolute maxima and minima of $f(x,y) = xy - 2x$ on the triangle with vertices $(0,0)$, $(4,0)$, $(0,4)$.

   (a) Sketch the region, shaded, on a set of xy-axes.

   (b) Find all interior critical points.

   (c) Find all critical points on the sides of the triangle along the x- and y-axes.

   (d) You now need to find any critical points on the third side of the triangle. Determine the $y = mx + b$ equation of the third side of the triangle. Substitute into $f(x,y) = xy - 2x$ to get it to be a function of only x, then find any critical points in the same way that you did for the other edges of the region.

   (e) Find the function values at all critical points and corners, organizing the results clearly. Conclude with a sentence giving the absolute maximum and minimum, and their locations.