#1 $f(x,y) = \frac{1}{3}x^3 + y^2 + 2xy - 6x - 3y + 4$ Assign 5/14/2018

$f_x(x,y) = x^2 + 2y - 6$ $\quad f_y(x,y) = 2y + 2x - 3 \Rightarrow 2y + 2x - 3 = 0$

Addition (Subtraction) Method! $\quad y = \frac{3-2x}{2}$

+1

$x^2 + 2y - 6 = 0$

$x^2 + 3 - 2x - 6 = 0$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = -1, 3$

$x = -1: y = \frac{5}{2}$ $\quad (-1, \frac{5}{2})$

$x = 3: y = -\frac{3}{2}$ $\quad (3, -\frac{3}{2})$

$f_{xx}(x,y) = 2x$ $\quad f_{yy}(x,y) = 2$ $\quad f_{xy} = 2$

$D = 4x - 4$

+2

At $(-1, \frac{5}{2})$, $D < 0$ so there is a saddle point at $(-1, \frac{5}{2})$

At $(3, -\frac{3}{2})$, $D > 0$ and $f_{xx} > 0$, so the function has a minimum at $(3, -\frac{3}{2})$.

$f(3, -\frac{3}{2}) = 9 + \frac{9}{4} - \frac{18}{2} - 18 + \frac{9}{2} + 4 = \frac{9}{4} - 5 = -\frac{29}{4}$

The function has a saddle point at $(-1, \frac{5}{2})$ and a minimum of $-\frac{29}{4}$ at $(3, -\frac{3}{2})$.

-7.25

#2 $z = x^2 + y^2 - 4x - 6y + 13$ on $0 \le x \le 3,\ 0 \le y \le 4$ Assign 5/14/2018

+1 $\begin{cases} f_x(x,y) = 2x - 4 \\ f_y(x,y) = 2y - 6 \end{cases}$ $\Longrightarrow$ Critical point at $(2,3)$, which is in region

+1

Edge $x=0$: $z = y^2 - 6y + 13$ $z' = 2y - 6 = 0$ when $y = 3$. Check $(0,3)$

Edge $x=3$: $z = y^2 - 6y + 10$ $z' = 2y - 6$ Check $(3,3)$

Edge $y=0$: $z = x^2 - 4x + 13$ $z' = 2x - 4$ Check $(2,0)$

Edge $y=4$: Check $(2,4)$

Corners: $(0,0)$, $(3,0)$, $(0,4)$, $(3,4)$

+1

Point	z
(2,3)	0
(0,3)	4
(3,3)	1
(2,0)	9
(2,4)	1
(0,0)	13
(3,0)	10
(0,4)	5
(3,4)	2

The function has a maximum of 13 at $(0,0)$ and a minimum of 0 at $(2,3)$.

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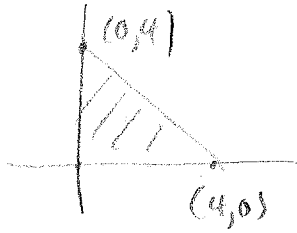
#3 $f(x,y) = xy - 2x$



$f_x(x,y) = y - 2$ Critical point $(0,2)$

$f_y(x,y) = x$

Boundary from $(0,0)$ to $(0,4) \Rightarrow x=0,\ f(x,y)=0$

Boundary from $(0,0)$ to $(4,0) \Rightarrow y=0,\ f(x,0) = -2x$

linear, no interior critical value.

Boundary from $(0,4)$ to $(4,0)$: $y = -x+4$

$$f(x,-x+4) = x(-x+4) - 2x$$
$$= -x^2 + 4x - 2x$$
$$= -x^2 + 2x = f(x)$$

$f'(x) = -2x + 2 \Rightarrow f'(x) = 0$ when $x = 1 \Rightarrow y = 3$

$(1,3)$

Check $f(0,0) = 0$ $\quad f(4,0) = -8$

$f(0,2) = 0$ $\quad f(1,3) = 3 - 2 = 1$

$f(0,4) = 0$

min of -8 at $(4,0)$

max of 1 at $(1,3)$