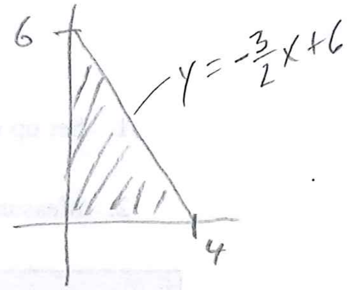
Math 254, Assignment 14 518

① $f(x,y) = xy - x - 2y$



$f_x(x,y) = y - 1 \Rightarrow y = 1$ $\boxed{\text{Critical point } (2,1)}$

$+\frac{1}{2}$ $f_y(x,y) = x - 2 \Rightarrow x = 2$

Edge $y=0$: $f(x,0) = -x \Rightarrow f'(x,0) = -1 \neq 0$ no critical point

$+\frac{1}{2}$ Edge $x=0$: $f(0,y) = -2y \Rightarrow f'(0,y) = -2 \neq 0$ no critical point

Edge $y = -\frac{3}{2}x + 6$: $f(x,y) = x\left(-\frac{3}{2}x+6\right) - x - 2\left(-\frac{3}{2}x+6\right)$

$= -\frac{3}{2}x^2 + 6x - x + 3x - 12$

$+1$ $= -\frac{3}{2}x^2 + 8x - 12$

$f'(x,y) = -3x + 8 = 0$ when $x = \frac{8}{3}$

$y = -\frac{3}{2}\left(\frac{8}{3}\right) + 6 = 2$

Check $\left(\frac{8}{3}, 2\right)$

(x,y)	$f(x,y)$
$(2,1)$	-2
$(0,0)$	0 max
$(0,6)$	-12 min
$(4,0)$	-4
$\left(\frac{8}{3}, 2\right)$	$\frac{8}{3} - 4 = \frac{8}{3} - \frac{12}{3} = -\frac{4}{3}$

$+\frac{1}{2}$

$+\frac{1}{2}$ The function has an absolute maximum of 0 at $(0,0)$, and an absolute minimum of -12 at $(0,6)$.

Math 254, Assignment 14, continued

(2) $f(x,y) = x^3 + y^2 - 6xy + 6x + 3y - 2$

$f_x(x,y) = 3x^2 - 6y + 6 = 0 \longrightarrow 3x^2 - 6\left(\frac{6x-3}{2}\right) + 6 = 0$

$3x^2 - 18x + 9 + 6 = 0$

$\times\frac{1}{2}$ $f_y(x,y) = 2y - 6x + 3 = 0 \Rightarrow y = \frac{6x-3}{2}$ $\quad 3x^2 - 18x + 15 = 0$

$3(x^2 - 6x + 5) = 0$

$3(x-1)(x-5) = 0$

$y = \frac{3}{2}, \frac{27}{2} \longleftarrow \boxed{x = 1, 5}$

$\times 1$ Critical points $\left(1, \frac{3}{2}\right), \left(5, \frac{27}{2}\right)$

$f_{xx}(x,y) = 6x \qquad f_{yy}(x,y) = 2 \qquad f_{xy}(x,y) = -6$

$D = 12x - 36$

$\times\frac{1}{2}$ $\left\{ \begin{array}{l} \left(1, \frac{3}{2}\right): D = 12 - 36 < 0 \\ \left(5, \frac{27}{2}\right): D = 60 - 36 > 0, \ f_{xx}\left(5, \frac{27}{2}\right) = 30 > 0 \text{ concave up at } \left(5, \frac{27}{2}\right) \end{array} \right.$

$f\left(5, \frac{27}{2}\right) = 125 + \frac{729}{4} - 405 + 30 + \frac{81}{2} - 2 = -29\frac{1}{4}$ $\pm\frac{1}{4}$

f has a saddle point at $\left(1, \frac{3}{2}\right)$ and a minimum of $-29\frac{1}{4}$ at $\left(5, \frac{27}{2}\right)$. $\pm\frac{1}{2}$

Math 254, Assignment 14, continued

(3) $P(3,5,2), Q(1,1,4), R(2,0,1)$

$+\frac{1}{2}$ idea

$\vec{PQ} = \langle -2,-4,2\rangle, \ \vec{PR} = \langle -1,-5,-1\rangle$

$$n = \vec{PQ}\times\vec{PR}: \begin{array}{ccccc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ -2 & -4 & 2 & -2 & -4 \\ -1 & -5 & -1 & -1 & -5 \end{array} = 4\vec{i} - 2\vec{j} + 10k - 4\vec{k} + 10\vec{i} - 2\vec{j}$$

$+\frac{1}{2}$

$\vec{n} = \langle 14,-4,6\rangle$ or $\langle 7,-2,3\rangle$

+2 total

$7x - 2y + 3z = d$

$7(1) - 2(1) + 3(4) = d$

$d = 17$

$\boxed{7x - 2y + 3z = 17}$

$+\frac{1}{2}$

$+\frac{1}{2}$