Math 254N ASSIGNMENT 17, SPRING 2018 **Due at 12:30 Friday, May 25th**

Suppose that water is flowing at a fairly slow speed through a cylindrical pipe of (inner) radius R. If we consider any point in a circular cross-section, the speed v of the water at that point (r, θ) of the cross-section is given by

$$v(r, \theta) = C(R^2 - r^2) \tag{1}$$

We'll measure the radius in inches and time in seconds, so the speed is in inches per second. This mathematical model for the flow of fluid through a pipe is valid at slow speeds, when the flow is what is called **laminar flow**. If the speed increases the flow becomes **turbulent**, and is more difficult to model mathematically.

1. Give the speed function for a pipe with a 10 inch inside **diameter**.

2. What is the speed of the water at the inside surface of the pipe? (Show how you obtain your answer.)

3. What is the speed of the water at the center of the pipe? (Again, show how you obtain it. Your answer will be in terms of C.)

If we do a polar integral over the cross-section, the area elements are given by $r\,dr\,d\theta$, and are in square inches. Thus the units of $v(r, \theta)\,r\,dr\,d\theta$ are in/sec $\times$ in$^2 =$ cubic inches per second. (Radians are actually unitless, so $d\theta$ has no units.) The integral then measures the total volume of water passing through such a cross-section in one second. The following illustrates how we could determine the constant C in equation (1) experimentally. **Round to four significant figures throughout the following.**

4. By the above discussion, the number of cubic inches of water passing through any cross-section of the pipe each second can also be calculated by integrating (1) over a cross-section of the pipe. Do that, obtaining an answer in terms of the unknown constant C.

5. Based on your previous answer, how many cubic inches of water will pass through a cross-section in one minute?

6. You let water run through the pipe for 1 minute, collecting the output, which turns out to be 78.6 gallons (US, liquid). Use an online converter to convert this to cubic inches.

7. Use your answers to Exercises 5 and 6 to determine the value of C.

8. The average speed of the fluid in the pipe is the flow rate found in Exercise 4 divided by the cross-sectional area of the pipe. What is the average speed?

9. How does the average speed compare with the maximum speed, which occurs in the center of the pipe? Answer with a brief sentence.