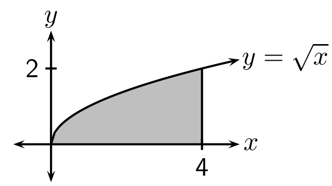
Math 254N Assignment 18, Spring 2018 **Due at 5:00 PM Monday, June 4th**

1. Suppose that the density at any point (x, y) in the shaded region shown to the right is given by $\rho(x, y) = 3x + y + 1$. Determine the coordinates of the center of mass $(\overline{x}, \overline{y})$ using the formulas



$$\overline{x} = \frac{\displaystyle\int\!\!\int_R x\,\rho(x, y)\,dx\,dy}{M}, \qquad \overline{y} = \frac{\displaystyle\int\!\!\int_R y\,\rho(x, y)\,dx\,dy}{M},$$

where

$$M = \int\!\!\int_R \rho(x, y)\,dx\,dy$$

and $\displaystyle\int\!\!\int_R$ means an iterated integral over the shaded region. You may use the Wolfram double integral calculator to compute integrals, but show your setup clearly. Use the standard notation $\overline{x}$ and $\overline{y}$ for the coordinates of the center of mass.

2. When the density of a plate is constant, the center of mass is often called the **centroid**, and it does not depend on the density. (Can you tell from the formulas why this is?) Therefore we can simply take $\rho(x, y) = 1$ in the above formulas to find the centroid. Find the centroid of the region shown to the right. **For this, compute by hand all three integrals needed** (one for $\overline{x}$, one for $\overline{y}$, and one for M).