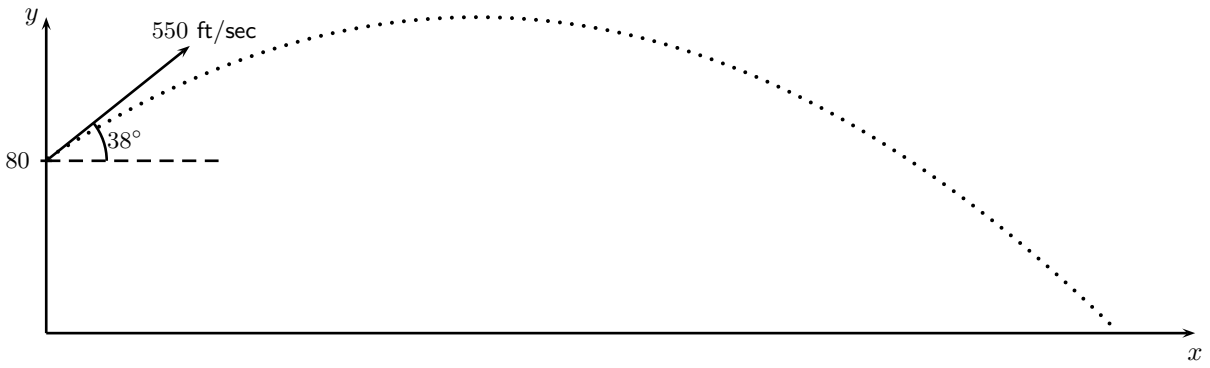
Math 254N ASSIGNMENT 6, SPRING 2018 **Due at the start of class 4/25**

This assignment concerns the following scenario: A projectile is launched from a height of 80 feet above level ground, and there is no force acting on it after it is launched, other than gravity. (Think of a cannonball from a cannon on a platform that raises the muzzle of the cannon to a height of eighty feet.) The cannonball is launched with an initial speed of 550 feet per second, at an angle of 38° above horizontal. It then travels in an arc, eventually hitting the ground. Even though this happens in three-dimensional space, the projectile travels in a vertical plane, so we can think of it as a two-dimensional situation. Here is a picture, with the dotted arc representing the path through the air:



We will ask several questions about the motion of the projectile:

- How far does the projectile travel horizontally before hitting the ground?
- How fast is the projectile going when it hits the ground? At what angle does it hit the ground?
- What is the maximum height of the projectile?
- How far does the projectile travel through the air (meaning distance along its path)?

1. The only force acting on the projectile during its flight is gravity, causing a downward acceleration of 32 feet per second per second (ft/sec^2). Thus the acceleration vector at any time t is $\vec{\mathbf{a}}(t) = \langle 0, -32 \rangle$. Our goal in this exercise is to find initial conditions $\vec{\mathbf{r}}(0)$ and $\vec{\mathbf{v}}(0)$.

   (a) $\vec{\mathbf{r}}(0)$ is simply the initial position, as a vector. This you should be able to get from the picture, or from thinking about the scenario.

   (b) $\vec{\mathbf{v}}(0)$ is the vector with magnitude 550 and angle of 38° to horizontal. Find the horizontal (x) and vertical (y) components by drawing a right triangle with a hypotenuse of 550 and using some trigonometry to determine the horizontal and vertical sides of the triangle, which will be the horizontal and vertical components of $\vec{\mathbf{v}}(0)$.

2. Now solve the initial value problem consisting of the acceleration $\vec{\mathbf{a}}(t) = \langle 0, -32 \rangle$ and the initial conditions $\vec{\mathbf{r}}(0)$ and $\vec{\mathbf{v}}(0)$ that you found in Exercise 1.

3. Note that when the projectile hits the ground the vertical component of position is zero. Use that fact to determine when the projectile hits the ground. Then use the horizontal component of position to determine the horizontal distance traveled before hitting the ground.

4. What is the vertical component of velocity of the object when it reaches its maximum height? Use that to determine the time when the maximum height is reached, and use that time to find the maximum height.