Math 254, Assignment 8

10½ points

123201

① $\vec{r}(t) = \langle 417t, -16t^2 + 351t + 250 \rangle$

$\vec{v}(t) = \langle 417, -32t + 351 \rangle$

a) 250 feet

b) $-16t^2 + 351t + 250 = 0 \Longrightarrow t = \dfrac{351 + \sqrt{(351)^2 + 4(16)(250)}}{32} = \dfrac{351 + 373.0965}{32}$

$= 22.6$ seconds

c) $-32t + 351 = 0$

$351 = 32t$

$t = 11.0$ seconds

(10.96875)

$h = -16(11)^2 + 351(11) + 250 = 2175$ feet

4111

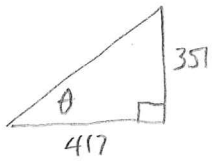
½ each

4 points

d) distance $= \displaystyle\int_0^{22.6} \sqrt{417^2 + (-32t + 351)^2}\, dt = 10,501$ feet

e) $d = 417(22.6) = 9424$ feet

f.)



$\theta = \tan^{-1}\left(\dfrac{351}{417}\right) = 40°$

173889

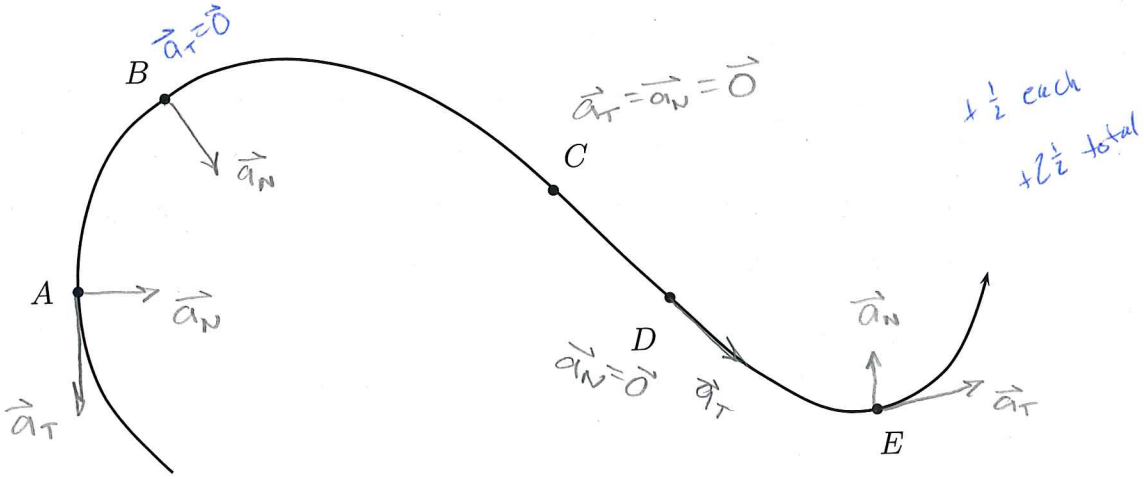
g) $\|\vec{v}(0)\| = \sqrt{417^2 + 351^2} = 545$ ft/sec

h) 417 ft/sec

Talk about this.

② Attached at back

2. A particle is traveling on the curve below, going from left to right overall, as indicated by the arrowhead at the end of the curve. At point A the particle is slowing down, at points B and C it is going at a constant speed, and at points D and E it is speeding up. **Assume that the curve is straight at points C and D.** Draw in the tangential and normal components of the acceleration at each of those points. Label each tangential component $\vec{\mathbf{a}}_T$ and label each normal component $\vec{\mathbf{a}}_N$. In cases where either is $\vec{\mathbf{0}}$, write that near the point.



+ ½ each
+2½ total

Math 254, Assn 8, continued

(3) $T(x,y,z) = 2xz^3 + y^2 + 3yz$

a) $$\frac{T(2,1,3) - T(0,2,1)}{\sqrt{(2-0)^2+(1-2)^2+(3-1)^2}} = \frac{\overset{108}{\overbrace{[2(2)(3)^3 + (1)^2 + 3(1)(3)]}} - [2(0)(1)^3 + 2^2 + 3(2)(1)]}{3}$$

+1 +¼ +¼ +¼

$$= \frac{118-10}{3} = \frac{108}{3} = 36\,°C/cm$$

+¼ total

b) $T_x(x,y,z) = 2z^3$ $\quad \frac{\partial T}{\partial y} = 2y + 3z$

+1 $$\frac{\partial^2 T}{\partial x \partial y} = 0 \qquad T_{xx}(x,y,z) = 0$$

c) $$\frac{\partial T}{\partial z} = 6xz^2 + 3y \qquad T_z(2,2,1) = 6(2)(1)^2 + 3(2) = 18\,°C/cm$$

+½

(4) a) $$\frac{T(2,2,1.5) - T(2,2,1)}{1.5 - 1} = \frac{26.5 - 14}{0.5} = 25\,°C/cm$$

+½ each b) $$\frac{T(2,2,1.1) - T(2,2,1)}{1.1 - 1} = \frac{15.924 - 14}{0.1} = 19.24\,°C/cm$$

c) $$\frac{T(2,2,1.01) - T(2,2,1)}{1.01 - 1} = \frac{14.181204 - 14}{0.01} = 18.1204\,°C/cm$$