At several points in this assignment you will be asked to compute average rates of change of a function $f$ from one point $\left(x_{1}, y_{1}\right)$ to another point $\left(x_{2}, y_{2}\right)$. For each you should show your work as follows:

$$
\begin{equation*}
\text { Ave ROC }=\frac{f\left(x_{2}, y_{2}\right)-f\left(x_{1}, y_{1}\right)}{\text { distance }}=\frac{\text { Value } 2-\text { Value } 1}{\text { Value of distance }}=\text { Answer, with units! } \tag{1}
\end{equation*}
$$

$x_{1}, y_{1}, x_{2}$ and $y_{2}$ should all be the appropriate numbers, and instead of $f$ you should use the letter for whatever function you are working with. Distance in the first fraction should be an expression indicating how you will get the distance. (It will always be either a subtraction or an application of the Pythagorean Theorem.)

1. The level curves below represent temperatures $T$ on a rectangular sheet of metal. The temperatures are in degrees Fahrenheit, and the distances are in feet (with $x$-values being on the horizontal axis and $y$-values on the vertical, as usual). Use the plot to answer the questions below it; give units with all answers for which they are appropriate
Answers will be approximate, but should be reasonably close.

(a) Give the temperature at $(7,3)$, using the notation $T(x, y)$. (In other words, write " $T(7,3)=\ldots$ ".
(b) Suppose that an ant is walking across the sheet of metal from $(8,0)$ to $(8,4)$. Compute the average rate of change, showing your work (as described in (1) above). Round your answer to the neraest tenth..
(c) Suppose that east is in the positive $x$-direction and north is in the positive $y$-direction. In a sentence, describe (qualitatively, and with the temperature at $A$ thrown in) what the ant is feeling as it walks from northwest to southeast through point A. ("Through" means including a bit on either side.)
(d) Describe (as in the previous part, with a sentence) what the ant is feeling as it walks from southwest to northeast through point $A$.
(e) Find the average rate of change in temperature from $(7,1)$ to $(7,5)$. Show your work as described in(1).
(f) Restate your answer to (e) in a sentence that mentions all the relevant values and includes one of the words increasing or decreasing. It should go something like "As one travels from $(7,1)$ to ..."
(g) Find the average rate of change in temperature from $(5,6)$ to $(9,1)$, then write a sentence interpreting it.

Actual Temperature ( ${ }^{\circ} \mathrm{F}$ )
2. The table to the right gives the wind-chill temperature $W$ (in degrees Fahrenheit) for various actual temperature/wind speed combinations (in that order). Wind speed is in miles per hour and actual temperature is given in degrees Fahrenheit. Use it to answer the questions below. (Table from the National Weather Service.)

| Actual Temperature ( ${ }^{\circ} \mathrm{F}$ ) |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\frac{\overline{\mathrm{O}}}{\frac{\mathrm{I}}{\xi}}$ | $\bar{v} V^{T}$ | -10 | 0 | 10 | 20 | 30 | 40 |
|  | 5 | -22 | -11 | 1 | 13 | 25 | 36 |
| $\begin{aligned} & \stackrel{\rightharpoonup}{0} \\ & \stackrel{\otimes}{0} \\ & \tilde{0} \\ & 0 \\ & i=1 \end{aligned}$ | 10 | -28 | -16 | -4 | 9 | 21 | 34 |
|  | 15 | -32 | -19 | -7 | 6 | 19 | 32 |
|  | 20 | -35 | -22 | -9 | 4 | 17 | 30 |

(a) In a sentence, describe the relationship between wind-chill and wind speed. Your sentence should be something like "As wind speed increases, ...".
(b) Describe the relationship between wind-chill and actual temperature.
(c) Give the value of $W(10,15)$, with units. Then write a short sentence telling what this tells us. (Your sentence should mention three numbers, with their units!)
(d) Find the rate of change in wind-chill from a wind speed/actual temperature combination of $0^{\circ} \mathrm{F}$ and 10 mph to $30^{\circ} \mathrm{F}$ and 10 mph .
(e) Find the rate of change in wind-chill from a wind speed/actual temperature combination of $20^{\circ} \mathrm{F}$ and 5 mph to $20^{\circ} \mathrm{F}$ and 20 mph .
3. In some location the temperature $T$ in degrees Fahrenheit of the ground at a depth of $d$ feet and at time $t$ days from some zero date is modeled by

$$
T(d, t)=30 e^{-0.2 d} \cos (0.0172 t-0.2 d)+60
$$

(a) Find the temperature at a depth of 3 feet, on the 200th day. Label your answer correctly using function notation.
(b) Give your answer to (a) as a sentence that makes it clear what both the question and answer are.
(c) Find the average rate of change of temperature at time fifty days, from a depth of 0.5 feet to a depth of 1.5 feet.
(d) Find the average rate of change of temperature at a depth of 2 feet, from zero days to half a year later. (Assume that there are 365 days in a year.)
(e) Write a sentence interpreting the mathematical statement $T_{t}(5,100)=-0.18$. Your sentence should include each of the three numbers you see, each with appropriate units, and one of the wrds increasing or decreasing.
(f) Write a sentence interpreting the mathematical statement $T_{d}(5,100)=-4.21$.
(g) Find $T_{t}(d, t)$ and $T_{d}(d, t)$. The second of these will require the product rule!

