Math 254N EXAM 1, SPRING 2018 **Name:** ______________________

Each numbered exercise is worth 6 points.

1. Give the parametric equations OR vector equation of the line containing $P(-1, 5, 2)$ and $Q(3, -2, 5)$.

$x = -1 + (3-(-1))t = -1 + 4t$
$y = 5 + (-2-5)t = 5 - 7t$
$z = 2 + (5-2)t = 2 + 3t$

OR

$\overrightarrow{PQ} = \langle 3-(-1), -2-5, 5-2 \rangle$
$= \langle 4, -7, 3 \rangle$
$\vec{x} = \overrightarrow{OP} + t\overrightarrow{PQ}$
$\boxed{\vec{x} = \langle -1, 5, 2 \rangle + t\langle 4, -7, 3 \rangle}$

2. Give the equation of the plane containing $P(-1, 5, 2)$ and having normal vector $\vec{\mathbf{n}} = \langle 3, -2, 5 \rangle$.

Looks like $ax + by + cz = d$, where $\langle a, b, c \rangle = \langle 3, -2, 5 \rangle$
$3x - 2y + 5z = d$
$3(-1) - 2(5) + 5(2) = d$
$-3 = d$
$\boxed{3x - 2y + 5z = -3}$

3. Give a vector $\vec{\mathbf{w}}$ of magnitude four in the direction of $\vec{\mathbf{u}} = \langle 1, -2, 2 \rangle$. **Give your answer as a "pure" vector, not a scalar times a vector.**

We divide $\vec{u}$ by its magnitude to get a unit vector (magnitude 1) and multiply by four:

$\vec{w} = 4\dfrac{\vec{u}}{\|\vec{u}\|} = 4\dfrac{\langle 1, -2, 2 \rangle}{\sqrt{1^2 + (-2)^2 + 2^2}} = 4\left\langle \frac{1}{3}, -\frac{2}{3}, \frac{2}{3} \right\rangle = \left\langle \frac{4}{3}, -\frac{8}{3}, \frac{8}{3} \right\rangle$

4. The sets of parametric equations below are some of the ones that we have been studying recently.

(a) Which sets describe motion with constant velocity? A

(b) Which sets describe motion with constant speed? A, C

(c) For which sets are the position and acceleration vectors parallel but opposite? C, D

A. $x = t - 2,\ y = 3t - 5$

B. $x = t^2 - 2,\ y = 3t^2 - 5$

C. $x = 4\sin\left(\frac{\pi}{2}t\right),\ y = 4\cos\left(\frac{\pi}{2}t\right)$

D. $x = 3\sin\left(\frac{\pi}{2}t\right),\ y = 5\cos\left(\frac{\pi}{2}t\right)$

5. An object is traveling along a path with parametric equations

$$x = t + 2\,, \qquad y = t^2 - 3t\,, \qquad t \geq 0$$

Give each of the following; **whenever it is possible to indicate how your answer is obtained, DO SO!** *3 points each part*

(a) The rectangular equation of the path, **simplified**.

$x = t + 2$

$t = x - 2$

$$y = (x-2)^2 - 3(x-2)$$
$$= (x-2)(x-2) - 3x + 6$$
$$= x^2 - 4x + 4 - 3x + 6$$
$$\boxed{y = x^2 - 7x + 10}$$

(b) The velocity at any time t. **Label your answer using correct notation.**

position vector: $\vec{r}(t) = \langle t+2, t^2 - 3t \rangle$) derivative

velocity vector: $\boxed{\vec{v}(t) = \langle 1, 2t - 3 \rangle}$

(c) The speed at five seconds, rounded to the nearest tenth. **Label your answer using correct notation.**

$$\|\vec{v}(5)\| = \|\langle 1, 2(5) - 3 \rangle\| = \|\langle 1, 7 \rangle\| = \sqrt{1^2 + 7^2} = \boxed{\sqrt{50}}$$

(d) The displacement from time one second to time three seconds.

$$\Delta\vec{r} = \vec{r}(3) - \vec{r}(1)$$
$$= \langle 5, 0 \rangle - \langle 3, -2 \rangle$$
$$= \boxed{\langle 2, 2 \rangle}$$

$\vec{r}(3) = \langle 3+2, 3^2 - 3(3) \rangle = \langle 5, 0 \rangle$

$\vec{r}(1) = \langle 1+2, 1^2 - 3(1) \rangle = \langle 3, -2 \rangle$

(e) The average velocity from time one second to time three seconds.

$$\text{Ave vel} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}(3) - \vec{r}(1)}{3 - 1} = \frac{\langle 2, 2 \rangle}{2} = \boxed{\langle 1, 1 \rangle}$$

Do **EXACTLY THREE** of the remaining six exercises. **Cross out the three that you do not want me to grade, and use additional paper if there is not room on the exam for all of your work.**

6. Let $\vec{\mathbf{v}} = \langle 2, -1 \rangle$ and $\vec{\mathbf{w}} = \langle 1, 3 \rangle$. Fill in the blanks below to give $\vec{\mathbf{v}}$ as the sum of two perpendicular vectors, one of which is parallel to $\vec{\mathbf{w}}$. Show any work you do in the space below.

$$\vec{\mathbf{v}} = \underline{\langle -\tfrac{1}{10}, -\tfrac{3}{10} \rangle} + \underline{\langle 2\tfrac{1}{10}, -\tfrac{7}{10} \rangle}$$

parallel to $\vec{w}$: $\text{proj}_{\vec{w}}\vec{v} = \dfrac{\vec{v}\cdot\vec{w}}{\vec{w}\cdot\vec{w}}\vec{w} = \dfrac{(2)(1)+(-1)(3)}{1^2+3^2}\langle 1,3 \rangle$

$$= -\tfrac{1}{10}\langle 1,3\rangle = \langle -\tfrac{1}{10}, -\tfrac{3}{10}\rangle$$

Other vector is $\text{perp}_{\vec{w}}\vec{v} = \vec{v} - \text{proj}_{\vec{w}}\vec{v}$

$$= \langle 2,-1\rangle - \langle -\tfrac{1}{10}, -\tfrac{3}{10}\rangle$$

$$= \langle 2\tfrac{1}{10}, -\tfrac{7}{10}\rangle$$

7. Give the vector equation of a line that

$\vec{x} = \vec{OP} + t\vec{PQ}$ → direction of line

→ point on line, like $(-5,-1,3)$, but as a vector

- is parallel to the plane with equation $x - 7y + 2z = 4$
- contains the point $P(-5,-1,3)$

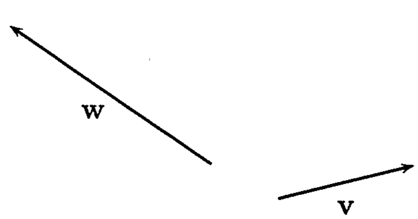
$\langle 1,-7,2\rangle = \vec{n}$ (normal to plane)

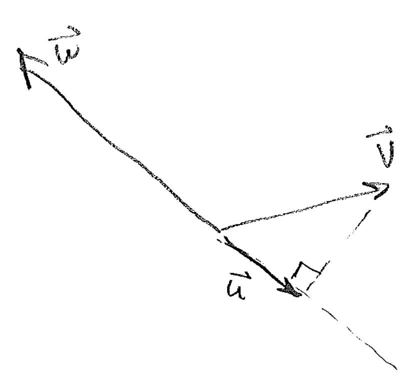
→ If line is parallel to the plane, its direction vector must be perpendicular to $\vec{n} = \langle 1,-7,2\rangle$

$\vec{x} = \langle -5,-1,3\rangle + t\langle$ any vector perp to $\rangle$

like $\vec{x} = \langle -5,-1,3\rangle + t\langle 7,1,0\rangle$

8. Two vectors $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$ are shown below and to the left. In the space below and to the right, draw and label the vector $\vec{\mathbf{u}} = \text{proj}_{\vec{\mathbf{w}}}\vec{\mathbf{v}}$, indicating clearly how you obtained it. **Indicate a right angle in the appropriate place and label the vector $\vec{\mathbf{u}}$.**





9. A particle traveling in the xy-plane has equations of motion $x = 3\sin \pi t$, $y = -3\cos \pi t$. Give the speed of the particle at any time t. **Simplify your answer as much as possible, showing clearly how it is done.** $\|\vec{v}\|$, $\vec{v} = r'$ so I need $\vec{r}$

$$\vec{r}(t) = \langle 3\sin\pi t, y = -3\cos\pi t\rangle$$

$$\vec{v}(t) = \langle 3\pi\cos\pi t, 3\pi\sin\pi t\rangle$$

$$\|\vec{v}(t)\| = \sqrt{(3\pi\cos\pi t)^2 + (3\pi\sin\pi t)^2}$$

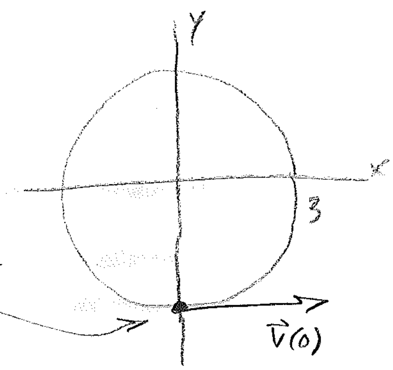
$$= \sqrt{9\pi^2\cos^2\pi t + 9\pi^2\sin^2\pi t}$$

$$= \sqrt{9\pi^2(\cos^2\pi t + \sin^2\pi t)} = \sqrt{9\pi^2} = 3\pi$$

10. For the particle from the previous exercise, determine whether the particle is traveling clockwise, or counterclockwise, around the circle. **To receive ANY credit for this exercise you must provide computations and an explanation of how they give your answer in the space below.**

Circle one: clockwise counterclockwise

The path is a circle of radius 3, centered at the origin:



$\vec{r}(0) = \langle 3\sin 0, -3\cos 0\rangle = \langle 0, -3\rangle$

$\vec{v}(0) = \langle 3\pi\cos 0, 3\pi\sin 0\rangle = \langle 3\pi, 0\rangle$

The position and velocity at time zero indicate the particle is traveling counterclockwise around the circle.

11. Determine the point where the two lines with parametric equations given below intersect.

Line 1: $x = 3 + t$, $y = 2 - 3t$, $z = -1 + 4t$ **Line 2:** $x = -1 - s$, $y = -4 + s$, $z = 1 - 2s$

Set x's and y's equal: $3+t = -1-s$, $2-3t = -4+s$ add

$5 - 2t = -5$

$-2t = -10$

$t = 5$

Solve for s: $3+5 = -1-s$

$9 = -s$

$s = -9$

$s=-9$ and $t=5$ give the same x and y for both lines. How about z?

$s=-9$: $z = 1-2(-9) = 19$

$t=5$: $z = -1+4(5) = 19$

So the lines intersect.

For $t=5$, $x=8$, $y=-13$. Point of intersection is $(8,-13,-19)$.