Math 254N EXAM 2, SPRING 2018 **Name:** 48 points

- Each numbered exercise is worth 6 points.
- For any answers that are obtained by a definite integral, set up the integral in a form that is ready for someone to enter into a calculator or other technology.
- Whenever units are given in an exercise, give appropriate units with your answer(s).
- **Do EXACTLY EIGHT** of the ten numbered exercises. **Cross out the two that you don't want me to grade.**

1. Solve the initial value problem $\vec{\mathbf{a}}(t) = \langle 10, 13e^{2t}\rangle$, $\vec{\mathbf{r}}(0) = \langle -2, 1\rangle$, $\vec{\mathbf{v}}(0) = \langle 4, 3\rangle$.

$\vec{v}(t) = \langle 10t + C_1, \frac{13}{2}e^{2t} + C_2\rangle$ $\quad$ $\vec{v}(0) = \langle C_1, \frac{13}{2} + C_2\rangle = \langle 4, 3\rangle$

$C_1 = 4, C_2 = -\frac{7}{2}$

$\vec{v}(t) = \langle 10t + 4, \frac{13}{2}e^{2t} - \frac{7}{2}\rangle$

$\vec{r}(t) = \langle 5t^2 + 4t + C_3, \frac{13}{4}e^{2t} - \frac{7}{2}t + C_4\rangle$ $\quad$ $\vec{r}(0) = \langle C_3, \frac{13}{4} + C_4\rangle = \langle -2, 1\rangle$

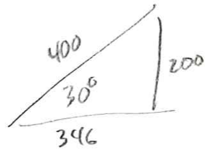
$C_3 = -2, C_4 = -\frac{9}{4}$

$$\boxed{\vec{r}(t) = \langle 5t^2 + 4t - 2, \frac{13}{4}e^{2t} - \frac{7}{2}t - \frac{9}{4}\rangle}$$

Integration +2
Finding constants +2
Idea +2

2. A projectile is launched from the edge of a 50 foot high building built on level ground, at an angle of $30°$ above horizontal and with an initial speed of 400 feet per second. Assuming the x-axis is along the ground and the point where the projectile is fired is on the y-axis, give the initial value problem to be solved for the equations of motion of the projectile. **Do not solve the initial value problem.**

$$\boxed{\vec{a}(t) = \langle 0, -32\rangle, \quad \vec{r}(0) = \langle 0, 50\rangle, \quad \vec{v}(0) = \langle 346, 200\rangle}$$



3. Bill the Human Cannonball is shot out of a cannon, to land in a pool of water. The cannon is on a raised platform, with the muzzle of the cannon at 20 feet above the ground, and the pool is at ground level. The equations of Bill's motion are

$$\vec{\mathbf{r}}\,(t) = \langle 75t, -16t^2 + 45t + 20\rangle, \qquad \vec{\mathbf{v}}\,(t) = \langle 75, -32t + 45\rangle\,, \qquad \vec{\mathbf{a}}\,(t) = \langle 0, -32\rangle$$

where distances are in feet and time is in seconds. **NOTE:** The solutions to $-16t^2+45t+20 = 0$ and $-32t + 45 = 0$ are $t = 3.2$ and $t = 1.4$, respectively.

(a) How far (horizontally) does Bill hope the center of the pool is from the muzzle of the cannon?

$$x(3.2) = 75(3.2) = 240 \text{ feet}$$

$+1\frac{1}{4}$ $+1\frac{1}{4}$ $+\frac{1}{2}$

(b) Give the actual distance (through the air) that the Bill travels before he lands in the pool.

$$d = \int_0^{3.2} \underbrace{\sqrt{75^2+(-32t+45)^2}}_{+1}\, dt$$

$+\frac{1}{2}$ $+\frac{1}{2}$

$+1$ for $\|\vec{v}\|$

4. Using the information from the previous exercise,

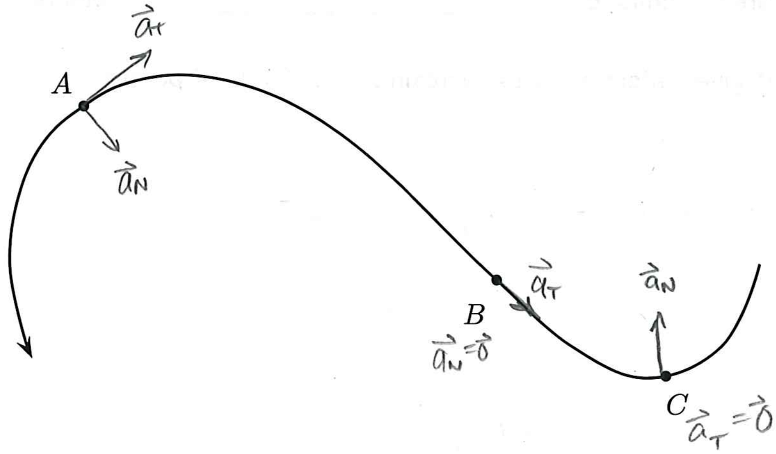
(a) What is the **displacement** from where Bill is launched to the apex (highest point) of his flight?

$$\Delta\vec{r} = \vec{r}(1.4) - \vec{r}(0) = \langle 105, 51.64\rangle - \langle 0, 20\rangle = \langle 105, 31.64\rangle$$

(b) How fast is Bill going when he is launched?

$$\|\vec{v}(0)\| = \|\langle 75, 45\rangle\| = 87 \text{ ft/sec}$$

5. (a) A particle is traveling on the curve below, **going from right to left overall**, as indicated by the arrowhead at the end of the curve. At points A and B the particle is speeding up, and at point C it is going at a constant speed. **Assume that the curve is straight at point B.** Draw in the tangential and normal components of the acceleration at each of those points. Label each tangential component $\vec{\mathbf{a}}_T$ and label each normal component $\vec{\mathbf{a}}_N$. In cases where either is $\vec{\mathbf{0}}$, write $\vec{\mathbf{a}}_T = \vec{\mathbf{0}}$ and/or $\vec{\mathbf{a}}_N = \vec{\mathbf{0}}$ near the point.



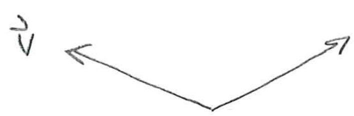
(b) A particle is traveling in a plane, with $\vec{\mathbf{v}}(3) = \langle -4, 1\rangle$ and $\vec{\mathbf{a}}(3) = \langle 3, 2\rangle$.

At time three the particle is (circle one from each row)

speeding up   slowing down   moving at constant speed

turning right   turning left   going straight

Give some indication in the space below how you got your answers. (There is a simple way to do this...)



6. The pressure in kilopascals (kPa) at the point (x, y, z), each in feet, is given by a function $P(x, y, z) = 2x + y^2 + z$. Determine the average rate of change from $(1, 0, 2)$ to $(2, 2, 4)$. **Show, using function notation, how you obtain your answer.**

$$\frac{P(2,2,4) - P(1,0,2)}{\sqrt{(2-1)^2 + (2-0)^2 + (4-2)^2}} = \frac{12-4}{\sqrt{9}} = \boxed{\frac{8}{3}\ \text{kPa/ft}}$$

+1 for ratio

+2 for distance

$+\frac{1}{2}$

7. The temperature on a plate of metal is a function $T(x,y)$ of the position (x,y) on the plate. Suppose that T is in degrees fahrenheit and x and y are in feet. Suppose also that we know

$$T(1,2) = 63.8, \qquad T_x(1,2) = 2.7, \qquad T_y(1,2) = -1.3$$

(a) What are the units for -1.3? $^{\circ}F/ft$ *2 points*

(b) Use the given information to approximate $T(3,1)$. *4 points*

$$T(3,1) = T(1,2) + T_x(1,2)(3-1) + T_y(1,2)(1-2)$$
$$= 63.8 + (2.7)(2) + (-1.3)(-1)$$
$$= 63.8 + 5.4 + 1.3$$
$$= 70.5^{\circ}F$$

8. For the information from the previous exercise, determine the rate of change passing through $(1,2)$ in the direction of the vector $\langle 4,3\rangle$.

$$\vec{u} = \frac{\langle 4,3\rangle}{\|\langle 4,3\rangle\|} = \left\langle \frac{4}{5}, \frac{3}{5} \right\rangle$$

$+\frac{1}{2}$

$$D_{\vec{u}}T(1,2) = \nabla T(1,2) \cdot \vec{u}$$
$$= \langle 2.7, -1.3\rangle \cdot \langle 0.8, 0.6\rangle$$
$$= 1.38^{\circ}F/ft$$

$+\frac{1}{2}$

9. The pressure in kilopascals (kPa) at the point (x, y, z), each in feet, is given by a function $P(x, y, z)$. It is known that

$$P(3,1,2) = 1031, \qquad P_x(3,1,2) = -1.3, \qquad P_y(3,1,2) = 0.8, \qquad P_z(3,1,2) = 2.4$$

Show any work you do for the following in the space below, and round the results of any computations to the hundredth's place.

(a) Give the maximum rate of increase of pressure when passing through $(3,1,2)$.

(b) Give a vector $\vec{\mathbf{v}}$ in the direction one would travel through $(3,1,2)$ in order to experience the maximum possible rate of increase of pressure.

max rate: 2.84 kPa/ft $\qquad\qquad$ $\vec{\mathbf{v}} = \langle -1.3, 0.8, 2.4 \rangle$

$\| \langle -1.3, 0.8, 2.4 \rangle \| = 2.84$

10. For $f(x, y) = 4e^{-3x} \sin(5x - 2y)$, find $f_x(x, y)$ and $f_y(x, y)$, **simplified.**

$$\boxed{\begin{aligned} f_x(x,y) &= 20e^{-3x}\cos(5x-2y) - 12e^{-3x}\sin(5x-2y) \\ f_y(x,y) &= -8e^{-3x}\cos(5x-2y) \end{aligned}}$$