The answers to all exercises are given at the bottom of the second page.

1. For each of the following, change the order of integration. It is suggested that you do this by first sketching the region in the xy-plane over which you are integrating, then setting up the integral in the opposite order.

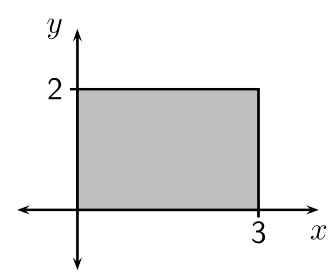
(a) $\displaystyle\int_0^1 \int_y^{\sqrt{y}} f(x,y)\, dx\, dy$ (b) $\displaystyle\int_0^2 \int_0^{4-y^2} f(x,y)\, dx\, dy$ (c) $\displaystyle\int_0^2 \int_1^{e^y} f(x,y)\, dx\, dy$

2. For each iterated integral are given below, determine whether it can be written as a product of two single integrals. If it can, write the two integrals - in such a case, your answer should look like $\iint f(x,y)dxdy = \left(\int g(x)dx\right)\left(\int h(y)dy\right)$. **Include all limits of integration and DO NOT compute any of the integrals.**

(a) $\displaystyle\int_0^{\pi} \int_1^5 x^2 \sin xy\, dx\, dy$ (b) $\displaystyle\int_1^4 \int_0^3 (3xy^2 + x)\, dx\, dy$ (c) $\displaystyle\int_1^2 \int_0^y x^2 y\, dx\, dy$

(d) $\displaystyle\int_0^1 \int_0^2 e^x e^{-y}\, dx\, dy$ (e) $\displaystyle\int_0^1 \int_0^2 \left(e^x + e^{-y}\right)\, dy\, dx$

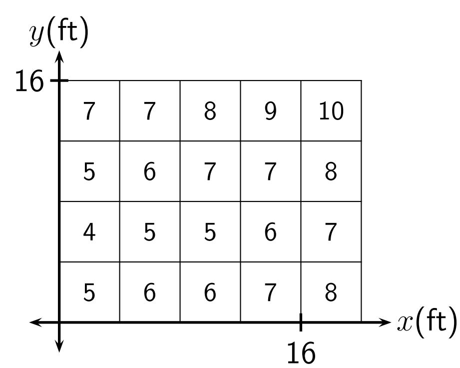
3. The graph to the right shows the cross-section of a duct through which air is flowing out toward you. The speed of the air at any point (x,y) in the duct is $v(x,y) = x + y$. The distance units are feet and the time units are seconds.



(a) Find the flow rate for the air through the duct; **give units with your answer**.

(b) Find the average speed of the air in the duct; **give units with your answer**.

4. The grid to the right is a map view of a 20 foot by 16 foot plot. The number in each square indicates the height (in feet) of the dirt at the center of that particular square, above some base level, given by a function $f(x,y)$. For each nested sum below, give an iterated integral that the sum approximates.



(a) $\big[(5+6+7+7)+(7+7+8+9)\big](4)(4)$

(b) $\big[(4+5+7)+(5+6+7)+(5+7+8)\big](4)(4)$

(c) $\big[(4+5+5+6+7)+(5+6+7+7+8)\big](4)(4)$

5. For each of the following, an iterated integral is given, along with the units for all variables. Give the units of the integral itself. **Radians are unitless!**

   (a) $\displaystyle\int_a^b \int_c^d \int_e^f \rho(x,y,z)\ dz\ dy\ dx$, x, y and z in centimeters, ρ in grams per cubic centimeter

   (b) $\displaystyle\int_0^{2\pi} \int_0^1 v(r,\theta)\ r\ dr\ d\theta$, r in feet, v in feet per second

   (c) $\displaystyle\int_a^b \int_c^d \int_e^f\ dz\ dy\ dx$, x, y and z in centimeters

   (d) $\displaystyle\int_a^b \int_c^d f(x,y)\ dy\ dx$, x, y and f in feet

   (e) $\displaystyle\frac{\int_a^b \int_c^d x\,\rho(x,y,z)\ dy\ dx}{\int_a^b \int_c^d \rho(x,y,z)\ dy\ dx}$, x and y in inches, ρ in grams per square inch

Answers

1. (a) $\displaystyle\int_0^1 \int_{x^2}^x f(x,y)\ dy\ dx$ (b) $\displaystyle\int_0^4 \int_0^{\sqrt{4-x}} f(x,y)\ dy\ dx$ (c) $\displaystyle\int_1^{e^2} \int_0^{\ln x} f(x,y)\ dy\ dx$

2. (a), (c), (e) can't be done. For (a) and (e), $f(x,y)$ can't be written as $g(x)h(y)$, and for (c) the region is not rectangular, because the upper limit of the inner integral is not constant.

   (b) $\displaystyle\int_1^4 \int_0^3 (3xy^2 + x)\ dy\ dx = \left(\int_1^4 x\ dx\right)\left(\int_0^2 (3y^2+1)\ dy\right)$

   (d) $\displaystyle\int_0^1 \int_0^2 e^x e^{-y}\ dx\ dy = \left(\int_0^2 e^x\ dx\right)\left(\int_0^1 e^{-y}\ dy\right)$

3. (a) The flow rate is $\displaystyle\int_0^3 \int_0^2 (x+y)\ dy\ dx = 15$ cubic feet per second.

   (b) The average speed is the flow rate divided by the area, which is $\frac{15}{6} = 2.5$ feet per second.

4. (a) $\displaystyle\int_8^{16} \int_0^{16} f(x,y)\ dx\ dy$ (b) $\displaystyle\int_0^{12} \int_4^{16} f(x,y)\ dy\ dx$ (c) $\displaystyle\int_4^{12} \int_0^{20} f(x,y)\ dx\ dy$

5. (a) grams (b) cubic feet per second (c) cubic centimeters
   (d) cubic feet (e) inches