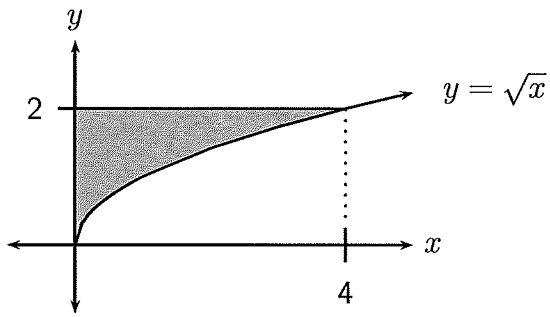
Math 254N EXAM 3, SPRING 2018 **Name:** ______________________________

Each numbered exercise is worth six points. Your score will be based on your best eight of the ten numbered exercises.

1. Fill in the limits of integration for the region shown to the right.

$$\int_{y=0}^{y=2}\int_{x=0}^{x=y^2} f(x,y)\;dx\;dy$$

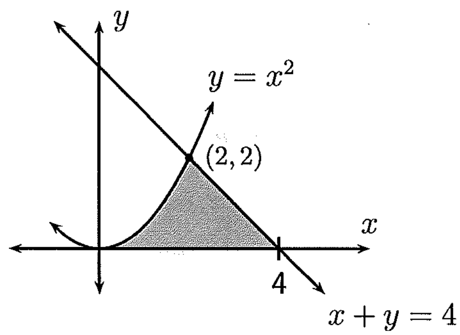


$$\int_{0}^{4}\int_{\sqrt{x}}^{2} f(x,y)\;dy\;dx$$

Do these first

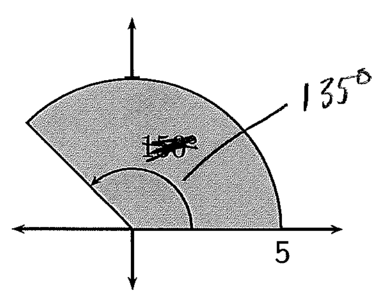
2. Fill in the limits of integration and the blanks for the region shown to the right, so that the single iterated integral gives the integral over the entire region.

$$\int_{0}^{2}\int_{\sqrt{y}}^{4-y} f(x,y)\;\underline{dx}\;\;\underline{dy}$$



3. Set up a polar integral to integrate a function $f(r,\theta)$ over the shaded region shown.

$$\int_{0}^{\frac{3\pi}{4}}\int_{0}^{5} f(r,\theta)\;\underline{r\,dr\,d\theta}$$



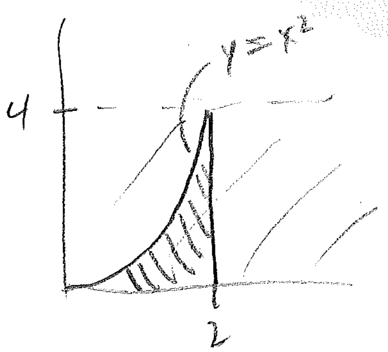
4. Consider the iterated integral $\int_0^4 \int_{\sqrt{y}}^2 f(x,y)\, dx\, dy$.

To the right, give the equivalent integral with the order of integration reversed.

$$\int_0^2 \int_0^{x^2} f(x,y)\, dy\, dx$$

$x = \sqrt{y}$

$y = x^2$



5. Find all critical points **for one** of the functions below, **showing clearly in the space below how you do it**.

(a) $f(x,y) = \frac{1}{3}y^3 + 4x^2y - 9y$, for which $f_x(x,y) = 8xy$ and $f_y(x,y) = y^2 + 4x^2 - 9$.

(b) $g(x,y) = \frac{1}{3}x^3 + \frac{1}{2}y^2 - xy - 6y$, for which $g_x(x,y) = x^2 - y$ and $g_y(x,y) = y - x - 6$.

$x^2 - y = 0 \qquad y - x - 6 = 0$

$x^2 = y \longrightarrow x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = -2, 3$

From $y = x^2$, when $x = -2, y = 4$
when $x = 3, y = 9$

$\boxed{(-2,4) \text{ and } (3,9)}$

$8xy = 0 \implies x = 0$ or $y = 0$

Now use $y^2 + 4x - 9 = 0$:

$x = 0$: $y^2 - 9 = 0 \implies y = -3, 3$ $\quad \boxed{(0,3), (0,-3)}$

$y = 0$: $4x^2 - 9 = 0$

$4x^2 = 9$

$x^2 = \frac{9}{4}$

$x = \pm\frac{3}{2}$ $\quad \boxed{(\frac{3}{2}, 0), (-\frac{3}{2}, 0)}$

6. For the function $f(x,y) = 4 + x^3 + y^3 - 3xy$ we have

$f_x(x,y) = 3x^2 - 3y\,,\quad f_y(x,y) = 3y^2 - 3x\,,\quad f_{xx}(x,y) = 6x\,,\quad f_{yy}(x,y) = 6y\,,\quad f_{xy}(x,y) = -3$

The critical points of the function are $(0,0)$ and $(1,1)$. Complete each sentence below to describe the behavior of the function at the given critical point (maximum, minimum, saddle point, etc.). **Include the value of any maxima or minima in your statement. Show work in the space below that supports your answers.**

At $(0,0)$ the function has a saddle point

At $(1,1)$ the function has a minimum of 3.

$D = (6x)(6y) - (-3)^2 = 36xy - 9$

At $(0,0)$, $D = -9 < 0$ so there is a saddle point

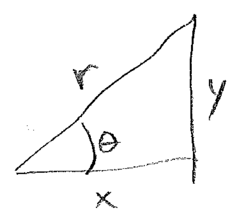
At $(1,1)$, $D = 36 - 9 = 27 > 0$ and $f_{xx}(1,1) = 6 > 0$

so the function has a minimum.

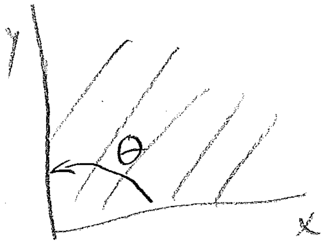
$f(1,1) = 4 + 1^3 + 1^3 - 3(1)(1) = 3$

7. Convert $\displaystyle\int_0^{\infty}\int_0^{\infty} e^{-x^2}e^{-y^2}\,dy\,dx$ into an equivalent polar iterated integral.

$e^{-x^2}e^{-y^2} = e^{-x^2-y^2} = e^{-(x^2+y^2)}$



$x^2+y^2=r^2$



$$\boxed{\int_0^{\frac{\pi}{2}}\int_0^{\infty} e^{-r^2}\, r\,dr\,d\theta}$$

r goes from 0 to ∞

θ goes from 0 to $\frac{\pi}{2}$

8. Consider the function $f(x,y) = x^2 + y^2 - 6x - 2y$ on the square region R with corners $(0,0)$, $(5,0)$, $(0,5)$ and $(5,5)$.

(a) Give any critical points in the interior of the region:

Show how you obtained them: $f_x(x,y) = 2x - 6 = 0$ when $x = 3$

$f_y(x,y) = 2y - 2 = 0$ when $y = 1$

Critical point $(3,1)$

(b) Give any critical point(s) on the edge of the region from $(0,5)$ to $(5,5)$:

Show how you obtained it (them): $y = 5$

$f(x,5) = x^2 - 6x + 15$

$f'(x,5) = 2x - 6 = 0$ when $x = 3$

Critical point $(3,5)$

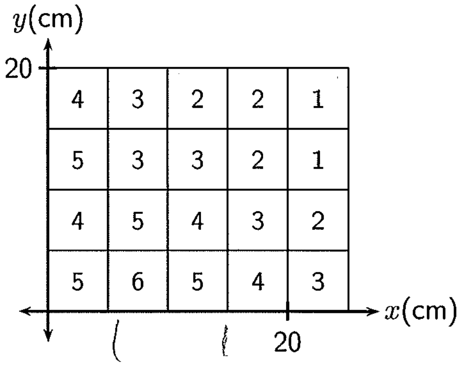
(c) Given that the absolute minima and maxima of the function occur in the interior of the region and/or on the edge of the boundary that you investigated (**including possibly its endpoints**), find them, and where they occur. **Show your work in the space below and fill in the blanks at the bottom.**

(x,y)	$f(x,y)$
$(0,5)$	15
$(3,5)$	6
$(5,5)$	10
$(3,1)$	-10

The absolute maximum value of the function on R is 15, and it occurs at $(0,5)$.

The absolute minimum value of the function on R is -10, and it occurs at $(3,1)$.

9. The grid to the right covers a 25 centimeter by 20 centimeter sheet of material. The number in each square indicates the area density, in grams (g) per square centimeter (cm) of the sheet at the center of that particular square, above some base level, given by the density function $\rho(x, y)$. For each iterated integral below, give a nested sum approximating the value of the integral. **DO NOT evaluate your sum, BUT give units with your answer.**



(a) $\int_0^{10} \int_{10}^{25} \rho(x, y)\, dx\, dy \approx$ $\left[(5+4+3)+(4+3+2)\right](5)(5)$ grams

(b) $\int_5^{15} \int_0^{20} \rho(x, y)\, dy\, dx \approx$ $\left[(6+5+3+3)+(5+4+3+2)\right](5)(5)$ grams

10. Four iterated integrals are given below. For each, determine whether it can be written as a product of two single integrals. If it can, write the two integrals to the right of the given integral - in such a case, your answer should look like $\iint f(x, y) dx dy = \left(\int g(x) dx\right) \left(\int h(y) dy\right)$. (Use the $\iint f(x, y) dx dy$ supplied, you provide the $\left(\int g(x) dx\right) \left(\int h(y) dy\right)$.) **Include the limits of integration and DO NOT compute any of the integrals.**

(a) $\int_0^{\pi} \int_1^5 x^2 \sin y\; dx\; dy$ $= \left(\int_1^5 x^2 dx\right)\left(\int_0^{\pi} \sin y\, dy\right)$

(b) $\int_0^1 \int_0^2 e^{x+y}\; dy\; dx$ $= \int_0^1 \int_0^2 e^x e^y\, dy\, dx = \left(\int_0^1 e^x dx\right)\left(\int_0^2 e^y dy\right)$

(c) $\int_0^1 \int_{x^2}^{x} (3xy + y)\; dy\; dx$ Can't do — variable limits

(d) $\int_1^4 \int_0^3 (3xy + y^3)\; dx\; dy$ Can't do — $3xy+y^3 \neq g(x)h(y)$