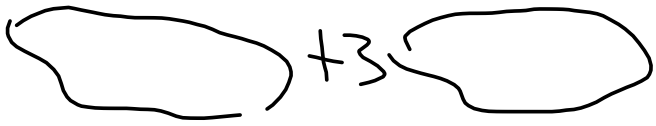
① $y = \frac{1}{2}e^{-3t} + \frac{1}{2}\sin t + \frac{3}{2}\cos t$

$y' = -\frac{3}{2}e^{-3t} + \frac{1}{2}\cos t - \frac{3}{2}\sin t$

$LHS = y' + 3y =$  $= 5\cos t$

$= RHS$

$y = \frac{1}{2}e^{-3t} + \frac{1}{2}\sin t + \frac{3}{2}\cos t$ is a sol to the ODE.

(2)

$$y = \frac{1}{2}e^{-3t} + \frac{1}{2}\sin t + \frac{3}{2}\cos t$$

$$y(0) = \frac{1}{2} + 0 + \frac{3}{2} = 2$$

(3)

$$y(0) = 1 - \frac{1}{25} \neq 1$$

So y is not a sol to the IVP.

④ $y(0) = C_2 + \frac{1}{13} = 0$

$C_2 = -\frac{1}{13}$

$y' = 2C_1\cos 2t - 2C_2\sin 2t - \frac{3}{13}e^{-3t}$

$y'(0) = 2C_1 - \frac{3}{13} = 1$

$2C_1 = \frac{16}{13}$

$C_1 = \frac{16}{26} = \frac{8}{13}$

(5) $y(0) = C_1 + C_2 - 1 = 1$

$C_1 = \frac{1}{4}$

$\frac{1}{4} + C_2 - 1 = 1$

$C_2 = 1\frac{3}{4} = \frac{7}{4}$

$y' = -3C_1e^{-3t} - C_2e^{-t} + \frac{1}{2}\cos t + \sin t$

$y'(0) = -3C_1 - C_2 + \frac{1}{2} = -2$

Add

$-2C_1 - \frac{1}{2} = -1$

$2C_1 + \frac{1}{2} = 1$

$2C_1 = \frac{1}{2}$

$C_1 = \frac{1}{4}$

$y(0)=0,\ y(10)=0$
$y'(0)=0,\ y'(10)=0$
} boundary values

ODE
IVP
BVP

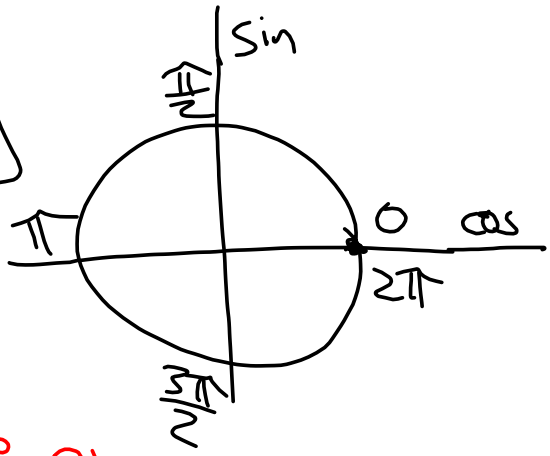
$y = C_1 \sin\frac{1}{2}x + C_2 \cos\frac{1}{2}x$

$y(\pi) = C_1 \sin\frac{\pi}{2} + C_2 \cos\frac{\pi}{2} = C_1 = 3$

$y' = \frac{1}{2} C_1 \cos\frac{1}{2}x - \frac{1}{2} C_2 \sin\frac{1}{2}x$

$y'(0) = \frac{1}{2} C_1 = -1$

$C_1 = -2$



The exercise is in error!