① Solve $\frac{1}{2}\ln\left|\frac{1}{2}y+5\right| = -3x+C_1$ for $y$, one step at a time. Show carefully what happens with constants, re-subscripting as you go.

② Is $y = x-1+Ce^{-x}$ a sol to $\frac{dy}{dx}+y=x$?

① Solve $\frac{1}{2}\ln\left|\frac{1}{2}y+5\right| = -3x + C_1$ for $y$

$$\ln\left|\frac{1}{2}y+5\right| = -6x + C_2 \quad (C_2 = 2C_1)$$

$$e^{\ln\left|\frac{1}{2}y+5\right|} = e^{-6x+C_2}$$

$x^a x^b = x^{a+b}$

$$\left|\frac{1}{2}y+5\right| = e^{-6x}e^{C_2}$$

$|x| = 5$

$x = \pm 5$

$$\left|\frac{1}{2}y+5\right| = C_3 e^{-6x} \quad (C_3 = e^{C_2})$$

$$\frac{1}{2}y+5 = \pm C_3 e^{-6x}$$

$$\frac{1}{2}y+5 = C_4 e^{-6x}$$

$$\frac{1}{2}y = C_4 e^{-6x} - 5$$

$$\boxed{y = C_5 e^{-6x} - 10} \quad (C_5 = 2C_4)$$

② Is $y = x - 1 + Ce^{-x}$ a sol to $\frac{dy}{dx} + y = x$?

$\frac{dy}{dx} = 1 - Ce^{-x}$

$LHS = \frac{dy}{dx} + y = \cancel{1} - \cancel{Ce^{-x}} + x \cancel{- 1} + \cancel{Ce^{-x}} = x = RHS.$

$y = x - 1 + Ce^{-x}$ is a sol to $\frac{dy}{dx} + y = x.$

$\frac{dy}{dx} + y = x$

$\frac{dy}{dx} = x - y$

$y = x$