① Can we solve $\frac{dy}{dx} - 3y = e^{5x}$ by separation of variables?

② Find the derivative of $x^2 e^{-3x}$.

③ Find the derivative (with respect to x) of ye^{-3x}, where y is an unknown function of x.

$$\frac{dy}{dx} - 3y = e^{5x}$$

not separable

linear

$$\frac{dy}{dx} = e^{5x} + 3y$$

$$dy = (e^{5x} + 3y)\,dx$$

$$\frac{dy}{dx} = g(x)h(y)$$

$$\underbrace{a_1(x)}\frac{dy}{dx} + a_0(x)y = f(x)$$

x^2e^{-3x}

$-3x^2e^{-3x}+2xe^{-3x}$

e^{9x}

$9e^{9x}$

ye^{-3x}

$-3ye^{-3x}+e^{-3x}\frac{dy}{dx}$

$e^{-3x}\frac{dy}{dx}-3e^{-3x}y$

$$\frac{dy}{dx} - 3y = e^{5x}$$

$$(ye^{-3x})' =$$

$$e^{-3x}\frac{dy}{dx} - 3e^{-3x}y = e^{5x}e^{-3x}$$

$$e^{-3x}\frac{dy}{dx} - 3e^{-3x}y$$

$$\frac{d(ye^{-3x})}{dx} = e^{2x}$$

$$\int dx = x + C$$

$$\int dt = t + C$$

$$\int d\,\big) = \big) + C$$

$$d(ye^{-3x}) = e^{2x}dx$$

$$\int d(ye^{-3x}) = \int e^{2x}dx$$

$$ye^{-3x} = \frac{1}{2}e^{2x} + C$$

$$ye^{-3x}e^{3x} = \left(\frac{1}{2}e^{2x} + C\right)e^{3x}$$

$$\boxed{y = \frac{1}{2}e^{5x} + Ce^{3x}}$$

$$\frac{dy}{dx} \underbrace{-3}_{p(x)} y = e^{5x}$$

$$p(x) = -3$$

$$u = \int p(x)\,dx$$

$$= \int -3\,dx$$

$$\boxed{u = -3x}$$

$$e^{u} = e^{-3x}$$

$$e^{-3x}\frac{dy}{dx} - 3e^{-3x}y = e^{5x}e^{-3x}$$

$$\frac{d\left[e^{-3x}y\right]}{dx} = e^{2x}$$

$$d\left[e^{-3x}y\right] = e^{2x}\,dx$$

$$e^{-3x}y = \frac{1}{2}e^{2x} + C$$

Multiply both sides by e^{3x}

$$y = \frac{1}{2}e^{2x}e^{3x} + Ce^{3x}$$

$$\boxed{y = \frac{1}{2}e^{5x} + Ce^{3x}}$$

$$\frac{dy}{dx} + \frac{1}{x} y = x^2 \qquad p(x)$$

$x > 0$, so $|x| = x$

$$p(x) = \frac{1}{x}$$

$$u = \int p(x)\,dx = \int \frac{1}{x}\,dx$$

$$= \ln|x|$$

$$= \ln x$$

$$e^u = e^{\ln x} = x$$

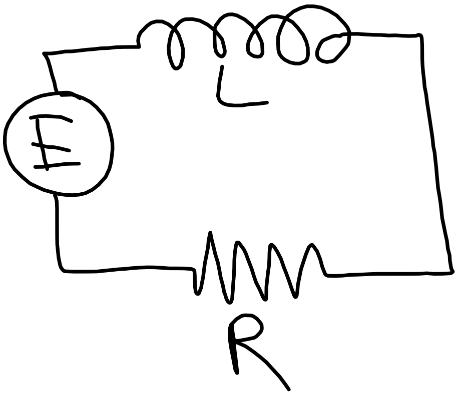
$$x\frac{dy}{dx} + x\frac{y}{x} = x^2 \cdot x$$

$$\frac{d[xy]}{dx} = x^3$$

$$d[xy] = x^3\,dx$$

$$xy = \frac{1}{4}x^4 + C$$

$$\boxed{y = \frac{1}{4}x^3 + \frac{C}{x}}$$



$$L\frac{di}{dt} + Ri = E(t)$$

$$2\frac{di}{dt} + 8i = 10\sin 2t$$

$$\frac{di}{dt} \underbrace{+4}_{p(t)} i = \underbrace{5\sin 2t}_{q(t)}$$

$$e^{4t}\frac{di}{dt} + 4e^{4t} i = 5e^{4t}\sin 2t$$