① Give a function $y=y(t)$ whose second derivative is -9 times the function. Then give another one

② Let r be a constant. Find a value of r for which $y=e^{rt}$ is a solution to

$r=-1,-2$

$$y''+3y'+2y=0$$

Note: $e^{rt}>0$ always!

$$r^2+3r+2=0$$

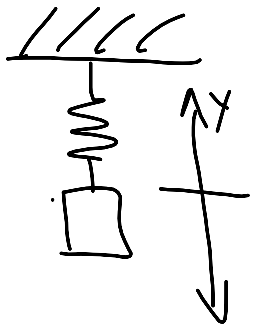
Auxiliary Equation

$$\boxed{y=C_1e^{-t}+C_2e^{-2t}}$$

$$ay'' + by' + cy = 0$$

2nd order
linear
constant coefficient
homogenous

ODE



$$F = ma = \boxed{m\frac{d^2y}{dt^2} = -ky}$$

$$\frac{d^2y}{dt^2} = -9y$$

Sol.: $\boxed{y = C_1 \sin 3t + C_2 \cos 3x}$

$$F = ma = \boxed{m\frac{d^2y}{dt^2} = -ky - \beta\frac{dy}{dt}}$$

"beta"
damping
constant
$(\beta > 0)$

$$m\frac{d^2y}{dt^2} + \beta\frac{dy}{dt} + ky = 0$$

$$\boxed{ay'' + by' + cy = 0}$$

$$y'' = -9y$$

$$y'' + 9y = 0$$

$$\boxed{y''+9y=0}$$

$$y = C_1 \sin 3t + C_2 \cos 3t$$

Assume $y = e^{rt}$

$y' = re^{rt}$

$y'' = r^2e^{rt}$

$$r^2e^{rt} + 9e^{rt} = 0$$

$$e^{rt}(r^2+9) = 0$$

$$\boxed{r^2+9=0}$$

$$r^2 = -9$$

$$r = \pm 3i$$

$(-3i)(-3i)$

$+9i^2$

-9

$$y = Ae^{3it} + Be^{-3it}$$

$$\boxed{\begin{array}{c} e^{i\theta} = \cos\theta + i\sin\theta \\ \text{Euler's Formula} \end{array}}$$

$$= A(\cos 3t + i\sin 3t) + B(\cos(-3t) + i\sin(-3t))$$

$$= A\cos 3t + Ai\sin 3t + B\cos 3t - Bi\sin 3t$$

$$= \underbrace{(A+B)}_{C_2}\cos 3t + \underbrace{(A-B)i}_{C_1}\sin 3t$$

$$\boxed{y = C_1 \sin 3t + C_2 \cos 3t}$$

$$y'' + 6y' + 9y = 0$$

$$r^2 + 6r + 9 = 0$$

$$(r+3)(r+3) = 0$$

$$r = -3$$

t

$$y = C_1 e^{-3t} + C_2 e^{-3t}$$

$$\boxed{y = C_1 e^{-3t} + C_2 t e^{-3t}}$$

$y'' + 10y' + 28y = 0 \longrightarrow$ See Sec, 3.1

$r^2 + 10r + 28 = 0$

$r = \dfrac{-10 \pm \sqrt{10^2 - 4(1)(28)}}{2(1)}$

$\sqrt{-12}$

$\sqrt{-1}\sqrt{4}\sqrt{3}$

$2i\sqrt{3}$

$= \dfrac{-10 \pm \sqrt{-12}}{2}$

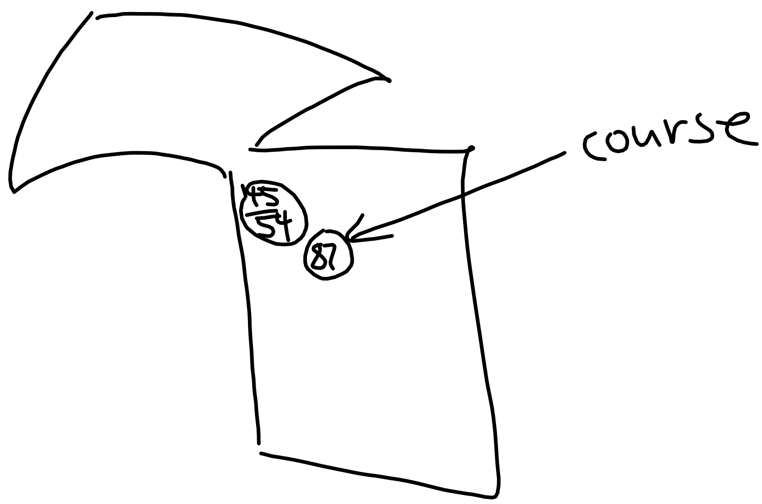
$= \dfrac{-10 \pm 2i\sqrt{3}}{2}$

$= \dfrac{-10}{2} \pm \dfrac{2i\sqrt{3}}{2}$ $a \pm bi$

$r = -5 \pm i\sqrt{3}$

$$\boxed{y = e^{-5t}\left(C_1 \sin\sqrt{3}\,t + C_2 \cos\sqrt{3}\,t\right)}$$

$y = C_1 e^{-5t} \sin\sqrt{3}t + C_2 e^{-5t} \cos\sqrt{3}t$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45
54
87