Solve $y'' + 4y = 5t^2$, $y(0) = 0$, $y'(0) = 1$

$r^2 + 4 = 0$

$r = \pm 2i$

$y_h = C_1 \sin 2t + C_2 \cos 2t$

Solve $y'' + 4y = 5t^2$, $y(0)=0$, $y'(0)=1$ (const)

Guess $y_p = At^2+Bt+C$

$y_p' = 2At+B$

$y_p'' = 2A$

$2A+4At^2+4Bt+4C = 5t^2$

$4At^2+4Bt+2A+4C = 5t^2$

$4A = 5$

$A = \frac{5}{4}$

$4B = 0$

$2A+4C = 0$

$2(\frac{5}{4})+4C = 0$

$\frac{5}{2}+4C = 0$

$4C = -\frac{5}{2}$

$C = -\frac{5}{8}$

$y_p = \frac{5}{4}t^2 - \frac{5}{8}$

Gen Sol: $y = C_1\sin 2t + C_2\cos 2t + \frac{5}{4}t^2 - \frac{5}{8}$

Solve $y'' + 4y = 5t^2$, $y(0) = 0$, $y'(0) = 1$

Gen Sol: $y = C_1 \sin 2t + C_2 \cos 2t + \frac{5}{4}t^2 - \frac{5}{8}$

$y' = 2C_1 \cos 2t - 2C_2 \sin 2t + \frac{5}{2}t$

$y(0) = 0 = C_2 - \frac{5}{8}$

$C_2 = \frac{5}{8}$

$y'(0) = 1 = 2C_1$

$C_1 = \frac{1}{2}$

$$y = \frac{1}{2}\sin 2t + \frac{5}{8}\cos 2t + \frac{5}{4}t^2 - \frac{5}{8}$$

$$y'' + 6y' + 5y = 5\cos 2t, \quad y(0) = 1, \quad y'(0) = 2$$

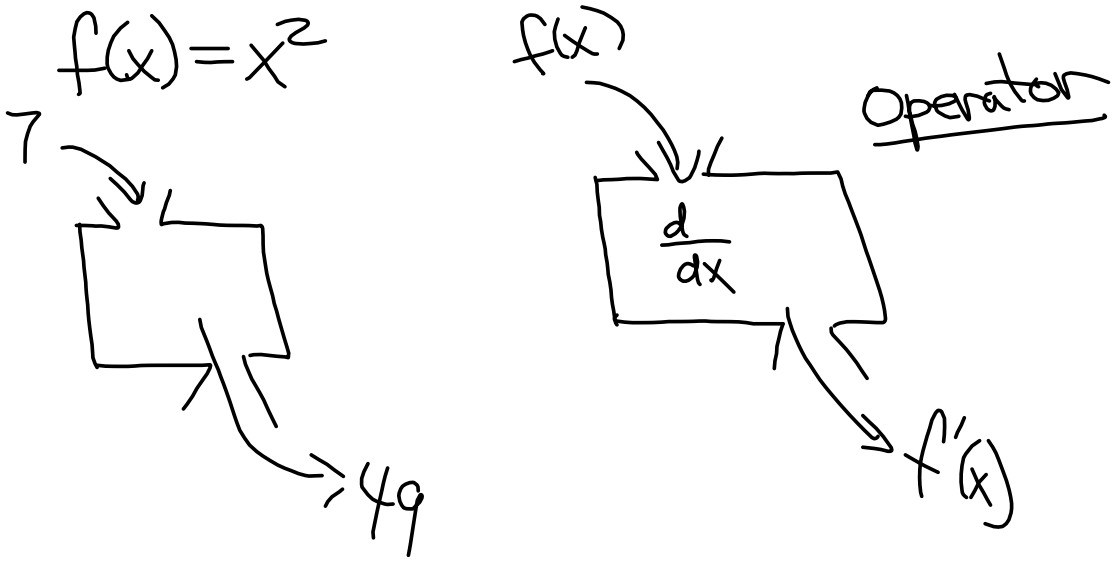
$$y = \frac{3}{2}e^{-t} - \frac{31}{58}e^{-5t} + \frac{12}{29}\sin 2t + \frac{1}{29}\cos 2t$$

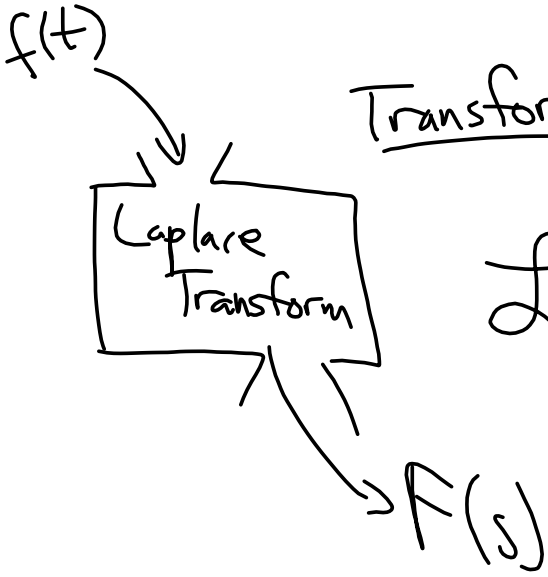
transient

steady-state

$b^2 - 4ac < 0$ underdamped

$b^2 - 4ac > 0$ overdamped


f(x) = x²
7
49
f(x)
Operator
d/dx
f'(x)



Transform

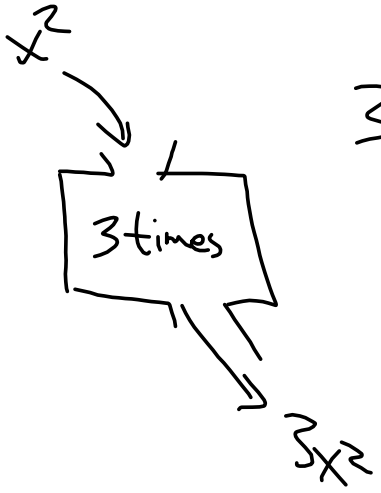
$$\mathcal{L}[f(t)] = F(s)$$

$$\frac{d}{dx}\left[c\,f(x)\right] = c\,\frac{d}{dx}\left[f(x)\right]$$

$$\frac{d}{dx}\left[f(x)+g(x)\right] = \frac{d}{dx}\left[f(x)\right]+\frac{d}{dx}\left[g(x)\right]$$

$\frac{d}{dx}$ is a linear operator

$$y = 5x^2 - 3x + 2$$



$$3\left[x^2+e^{-2x}\right] = 3x^2+3e^{-2x}$$

$$3\left[5x^2\right] = 5\left[3x^2\right]$$

$$\frac{d^2}{dx^2}+5\frac{d}{dx}-3$$