① Solve $y'' + 9y = \sin t$, $y(0) = 1$, $y'(0) = 1$

② Find the particular solution to $y'' + 9y = 2\cos 3t$

"Trick question?

① Solve $y'' + 9y = \sin t$, $y(0) = 1$, $y'(0) = 1$

$y'' + 9y = 0$

$r^2 + 9 = 0$

$r^2 = -9$

$r = \pm 3i$

$y_h = C_1 \sin 3t + C_2 \cos 3t$

Guess

$y_p = A \sin t + B \cos t$

$y_p' = A \cos t - B \sin t$

$y_p'' = -A \sin t - B \cos t$

$y_p'' + 9y = -A \sin t - B \cos t + 9A \sin t + 9B \cos t$

$= 8A \sin t + 8B \cos t = \sin t$

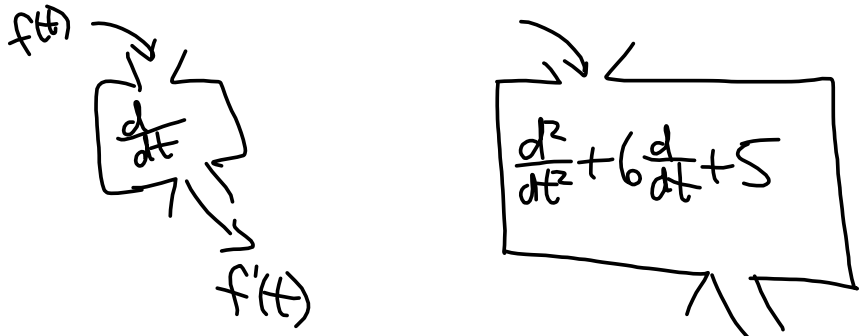
So $A = \frac{1}{8}$, $B = 0$

$y_p = \frac{1}{8} \sin t$

$y = C_1 \sin 3t + C_2 \cos 3t + \frac{1}{8} \sin t$

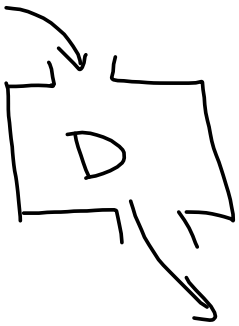
$\therefore$ use $y(0) = 1$, $y'(0) = 1$

$y = \frac{7}{24} \sin 3t + \cos 3t + \frac{1}{8} \sin t$



$$\frac{d}{dt}\left[f(t)+g(t)\right]=\frac{d}{dt}f(t)+\frac{d}{dt}g(t)$$

$$D = \frac{d^2}{dt^2} + 6\frac{d}{dt} + 5$$



If $y = y(t)$, then

$$D(y) = \frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 5y$$

$y = t^2 + 3t$

$D(y) = ?$

$$= 2 + 6(2t+3) + 5(t^2+3t)$$

$$= 2 + 12t + 18 + 5t^2 + 15t$$

$$= 5t^2 + 27t + 20$$

If $y = y(t)$, then

$$D(y) = \frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 5y \qquad x^2 = 9$$

$$D(c_1e^{-t} + c_2e^{-5t}) = 0$$

$$D\left(\frac{12}{29}\sin 2t + \frac{1}{29}\cos 2t\right) = 5\cos 2t$$

$$y'' + 9y = 2\cos 3t$$

Guess for $y_p = A\sin 3t + B\cos 3t$

doesn't "work"

Correct guess is $y_p = At\sin 3t + Bt\cos 3t$

$y'' + 9y = 2\cos 3t, \quad y(0) = 1, \quad y'(0) = 1$

Solution: $y = \frac{1}{3}\sin 3t + \cos 3t + \frac{1}{3}t\sin 3t$

$y_h = C_1 \sin 3t + C_2 \cos 3t$