① Give an ODE with the solution

$y = C_1 e^{-2t} + C_2 e^{-3t}$.

② Give an ODE with the solution

$y = C_1 \sin 2t + C_2 \cos 2t + 4e^{-3t}$

① Give an ODE with the solution

$y = C_1e^{-2t} + C_2e^{-3t}$ $\longrightarrow$ $y' = -2C_1e^{-2t} - 3C_2e^{-3t}$

$y'' =$ 

$y'' + 5y' + 6y = 0$

$r^2 + 5r + 6 = 0$

$(r+2)(r+3) = 0$

$r = -2, -3$

(2) Give an ODE with the solution

$$y = C_1 \sin 2t + C_2 \cos 2t + 4e^{-3t}$$

homogenous

$r = \pm 2i$

$r^2 = -4$

$r^2 + 4 = 0$

particular sol

$y = 4e^{-3t}$

$y' = -12e^{-3t}$

$y'' = 36e^{-3t}$

$y'' + 4y = ?$

$36e^{-3t} + 4(4e^{-3t}) = 52e^{-3t}$

$$\boxed{y'' + 4y = 52e^{-3t}}$$

Undamped, under-damped, over-damped, critically damped

a) $y'' + 3y' + 2y = 5\cos 2t$ (over-damped) $r^2 + 3r + 2 = 0$

b) $y'' + 4y = 5\cos 2t$ undamped

c) $y'' + 3y' + 2y = 3e^{-t}$ (overdamped)

d) $y'' + 6y' + 11y = t^2 + 4t$ (underdamped)

$r = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$

Example of critically damped: $y'' + 6y' + 9y = f(t)$

Guess for $y_p$?

a) $y'' + 3y' + 2y = 5\cos 2t$ $\quad y_p = A\sin 2t + B\cos 2t$

b) $y'' + 4y = 5\cos 2t$ $\quad y_h = C_1\sin 2t + C_2\cos 2t$ $\quad y_p = At\sin 2t + Bt\cos 2t$

c) $y'' + 3y' + 2y = 3e^{-t}$ $\quad y_p = Ae^{-t}$

d) $y'' + 6y' + 11y = t^2 + 4t$ $\quad y_p = At^2 + Bt + C$