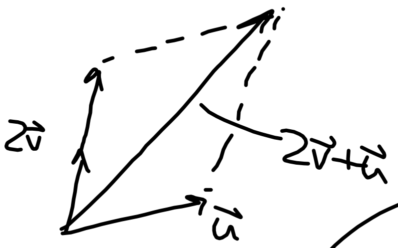
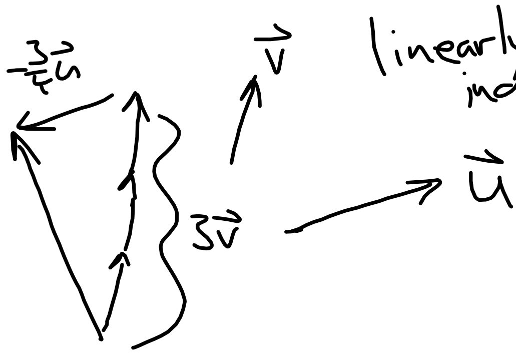
Guess $y=e^{rt}$

$$y''+3y'+2y=0$$
$$r^2+3r+2=0$$
$$(r+2)(r+1)=0$$
$$r=-1,-2$$
$$y=e^{-t},\ y=e^{-2t}$$
$$\boxed{y=C_1e^{-t}+C_2e^{-2t}}$$

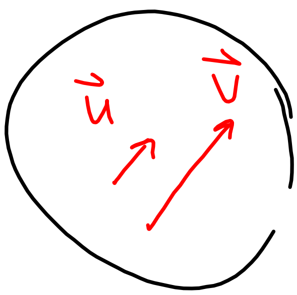
$$y''+4y'+4y=0$$
$$r^2+4r+4=0$$
$$(r+2)(r+2)=0$$
$$r=-2,-2$$
$$y=e^{-2t}$$
$$\boxed{y=C_1e^{-2t}+C_2te^{-2t}}$$

$2\vec{v} + \vec{u}$

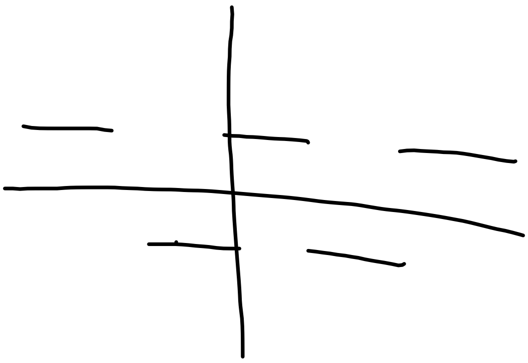




linearly indep



linearly dependent



Fourier series

$$f(t) = \overset{\cos 0t}{a_0} + a_1 \cos t + a_2 \cos 2t + \dots$$
$$b_1 \sin t + b_2 \sin 2t + \dots$$

$$y'' + 4y' + 4y = 0$$

$$y = C_1 e^{-2t} + C_2 e^{-2t}$$

W is the Wronskian

$f(t), g(t)$

$$\boxed{W = f(t)g'(t) - f'(t)g(t)}$$

$f(t) = e^{-t}$

$g(t) = e^{-2t}$

$$W = (e^{-t})(-2e^{-2t}) - (-e^{-t})(e^{-2t})$$

$$= -2e^{-3t} + e^{-3t}$$

$$= -1e^{-3t} \neq 0$$

f, g are indep

$$\boxed{W = f(t)g'(t) - f'(t)g(t)}$$

$f(t) = \sin t$
$g(t) = \cos t$

$$W = (\sin t)(-\sin t) - (\cos t)(\cos t)$$
$$= -\sin^2 t - \cos^2 t$$
$$= -(\sin^2 t + \cos^2 t)$$
$$= -1 \neq 0$$

$$y''+3y'+2y=0$$

Know $y_1=e^{-t}$ is a sol

Assume $\boxed{y_2=u(t)e^{-t}}$ is also a sol, u is unknown

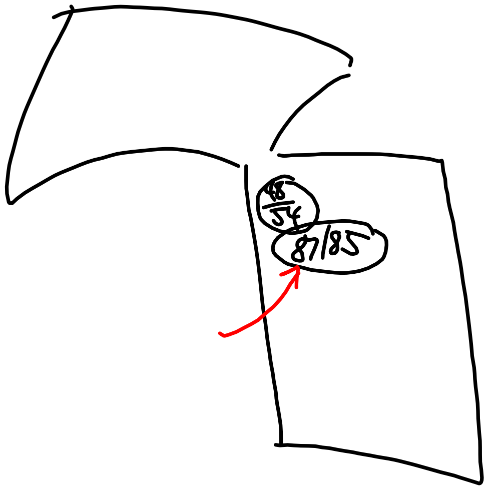
$$y_2'=u(-e^{-t})+u'e^{-t}$$
$$=-ue^{-t}+u'e^{-t}$$
$$=e^{-t}(-u+u')$$

$$y_2''=ue^{-t}-u'e^{-t}-u'e^{-t}+u''e^{-t}$$
$$=ue^{-t}-2u'e^{-t}+u''e^{-t}$$
$$=e^{-t}(u-2u'+u'')$$

$$LHS=e^{-t}(u-2u'+u'')+3e^{-t}(-u+u')+2ue^{-t}$$
$$=e^{-t}[u-2u'+u''-3u+3u'+2u]$$
$$=e^{-t}[u''+u']=0$$

Let $v(t)=u'(t)$, $v'(t)=u''$

$$\left.\begin{aligned} u''+u'&=0\\ v'+v&=0\end{aligned}\right\}\text{Reduction of order}$$


$\frac{48}{54}$
87/85