Let $y = ue^{-t}$, where u is a function of t

a) Find y' and y'' (product rule!)

b) Substitute into $y'' + 3y' + 2y = 0$

$y_2 = u\underbrace{e^{-t}}_{y_1}$ $\quad y' = -ue^{-t} + u'e^{-t}$

$y'' = ue^{-t} - u'e^{-t} - u'e^{-t} + u''e^{-t}$

$= ue^{-t} - 2u'e^{-t} + u''e^{-t}$

$y'' + 3y + 2y = 0$

Know $y_1 = e^{-t}$ is a solution. Find another sol y_2.

$$\begin{aligned} y'' + 3y' + 2y = \; & ue^{-t} - 2u'e^{-t} + u''e^{-t} \\ & -3ue^{-t} + 3u'e^{-t} \\ & +2ue^{-t} \\ \hline & = 0 + u'e^{-t} + u''e^{-t} = 0 \end{aligned}$$

$$\underbrace{e^{-t}}_{\text{not zero}}(\underbrace{u'' + u'}_{\text{must be zero}}) = 0$$

$u'' + u' = 0$

$u = u(t)$

Let $v(t) = u'(t)$

$v = u'$

$v' = u''$

$v' + v = 0$

$\frac{dv}{dt} + v = 0 \xrightarrow{\text{could}} \frac{dv}{dt} = v$

instead

$dv = v\,dt$

$\frac{dv}{v} = dt$

$\frac{dv}{dt} = -v$

$v = e^{-t}$

$u' = e^{-t}$

$u = -e^{-t}$

not really needed

$y_2 = u\underbrace{e^{-t}}_{y_1} = -e^{-t}e^{-t} = \underbrace{-e^{-2t}}_{\text{really } y_2}$

General sol is $y = C_1e^{-t} + C_2e^{-2t}$

$$\frac{d^4y}{dx^4}=12 \qquad y(0)=y(12)=y'(12)=y''(0)=0$$

$$\frac{d^3y}{dx^3}=12x+C_1$$

$$\frac{d^2y}{dx^2}=6x^2+C_1x+C_2 \xrightarrow{y''(0)=0} 0=C_2$$

$$\frac{d^2y}{dx^2}=6x^2+C_1x$$

$$\frac{dy}{dx}=2x^3+\frac{1}{2}C_1x^2+C_3 \xrightarrow{y'(12)=0} 0=\overset{2(12)^3}{3456}+\frac{1}{2}C_1(144)+C_3$$

$$\boxed{72C_1+C_3=-3456}$$

$$y=\frac{1}{2}x^4+\frac{1}{6}C_1x^3+C_3x+\cancel{C_4}\to 0$$

because $y(0)=0$

$$\xrightarrow{y(12)=0} 0=\frac{1}{2}(12)^{\cancel{4}\,3}+\frac{1}{6}C_1(12)^{\cancel{3}\,2}+\cancel{12}C_3$$

$$=864+24C_1+C_3$$

$$\boxed{24C_1+C_3=-864}$$

Subtract

$$48C_1=-2592$$

$$C_1=-54$$

$$C_3=-864+(24)(54)$$

$$C_3=432$$

$$\boxed{y=\frac{1}{2}x^4-9x^3+432x}$$

Turn In: 4.2: 1 both parts!

Solve $\frac{d^4y}{dx^4} = 24$

$y(0) = y(10) = y''(0) = y''(10) = 0$