Use an integrating factor (see fmla sheet and help each other) to solve

$$\frac{dy}{dt} + 3y = 2\sin 5t$$

$$\frac{dy}{dt}+3y=2\sin 5t \qquad u=\int 3\,dt=3t$$

$$e^{3t}\frac{dy}{dt}+3ye^{3t}=2e^{3t}\sin 5t \qquad e^{u}=e^{3t}$$

$$\frac{d(e^{3t}y)}{dt}=2e^{3t}\sin 5t \qquad \frac{dy}{dx}+p(x)y=q(x)$$

$$d(e^{3t}y)=2e^{3t}\sin 5t\,dt$$

$$\int d(e^{3t}y)=2\int e^{3t}\sin 5t\,dt$$

$$e^{3t}y=\frac{2e^{3t}}{3^2+5^2}(3\sin 5t-5\cos 5t)+C$$

$$e^{-3t}e^{3t}y=e^{-3t}\frac{2e^{3t}}{34}(3\sin 5t-5\cos 5t)+Ce^{-3t}$$

$$\boxed{y=\frac{1}{17}(3\sin 5t-5\cos 5t)+Ce^{-3t}}$$

$$\frac{25}{25}\int e^{3t}\sin 5t\,dt = -\frac{1}{5}e^{3t}\cos 5t + \frac{3}{25}e^{3t}\sin 5t - \frac{9}{25}\int e^{3t}\sin 5t\,dt$$

$$+\frac{9}{25}\int e^{3t}\sin 5t\,dt \qquad\qquad +\frac{9}{25}\int e^{3t}\sin 5t\,dt$$

$$\frac{34}{25}\int e^{3t}\sin 5t\,dt = -\frac{1}{5}e^{3t}\cos 5t + \frac{3}{25}e^{3t}\sin 5t$$

$$\int e^{3t}\sin 5t\,dt = -\frac{5}{34}e^{3t}\cos 5t + \frac{3}{34}e^{3t}\sin 5t$$

$$\frac{dy}{dt}+3y=2\sin 5t$$

Guess

$$y_p=A\sin 5t+B\cos 5t$$

$$y'=5A\cos 5t-5B\sin 5t$$

$$\frac{dy}{dt}+3y=0$$

$$r+3=0$$

$$r=-3$$

$$y_h=Ce^{-3t}$$

$$5A\cos 5t-5B\sin 5t+3A\sin 5t+3B\cos 5t=2\sin 5t$$

$$\underbrace{(3A-5B)}_{=2}\sin 5t+\underbrace{(5A+3B)}_{=0}\cos 5t=2\sin 5t$$

$$3A-5B=2 \Longrightarrow 9A-15B=6$$

$$5A+3B=0 \Longrightarrow 25A+15B=0$$

$$34A=6$$

$$A=\frac{6}{34}=\frac{3}{17}$$

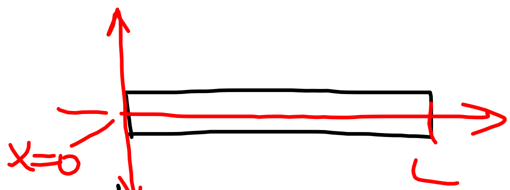
$$\frac{30}{34}+3B=0$$

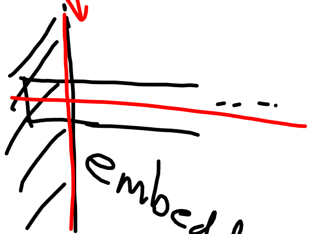
$$3B=-\frac{30}{34}$$

$$B=-\frac{10}{34}=-\frac{5}{17}$$

$$\boxed{y_p=\frac{3}{17}\sin 5t-\frac{5}{17}\cos 5t}$$

$$\boxed{y=y_h+y_p=\frac{3}{17}\sin 5t-\frac{5}{17}\cos 5t+Ce^{-3t}}$$

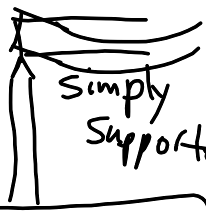




embedded

$y(0)=0$

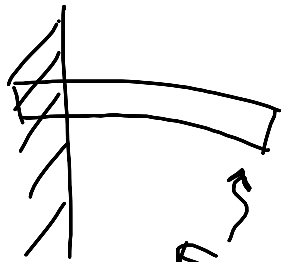
$y'(0)=0$



simply supported

$y(0)=0$

$y''(0)=0$



Free end

$y''(0)=0$

$y'''(0)=0$

$$\underbrace{E}_{\text{Modulus of Elasticity}}\ \underbrace{I}_{\text{cross-sectional moment of inertia}}\frac{d^4y}{dx^4}=\underbrace{w(x)}_{\text{linear weight density (constant for us)}}$$

$$\frac{d^4y}{dx^4}=\frac{w(x)}{\underbrace{EI}_{\text{number}}}$$

$$\frac{d^4y}{dx^4}=24$$

$$\frac{d^3y}{dx^3}=24x+C_1$$