1. $y(0) = y(10) = 0$ $y = y(x)$

$y'(10) = 0$

$y''(0) = 0$

$$\overset{y}{\cancel{p(x)}} = x^2 - 5x + 2$$

$$p'(x) = 2x - 5$$

$$p''(x) = 2$$

$$L[p(x)] = (x^2-1)2 + 2x(2x-5)$$

$$= 2x^2 - 2 + 4x^2 - 10x$$

$$= 6x^2 - 10x - 2$$

$p(x) = x^2 - 5x + 2$

$$L[q(x)] = 60x^3 - 36x$$

$q(x) = 5x^3 - 3x$

$$= 12q(x)$$

$q(x)$ is an eigenfunction of L with eigenvalue 12.

$$\underbrace{y'' + \frac{1}{4}y = 0}, \quad y(0) = 3, \; y(\pi) = -4$$

$$r^2 + \frac{1}{4} = 0$$

$$r^2 = -\frac{1}{4}$$

$$r = \pm\frac{1}{2}i$$

$$y = C_1 \sin\frac{1}{2}x + C_2 \cos\frac{1}{2}x$$

$$3 = \cancelto{0}{C_1 \sin 0} + \cancelto{1}{C_2 \cos 0} \Rightarrow C_2 = 3$$

$$-4 = \cancelto{1}{C_1 \sin\frac{\pi}{2}} + \cancelto{0}{C_2 \cos\frac{\pi}{2}} \Rightarrow C_1 = -4$$

$$\boxed{y = -4\sin\frac{1}{2}x + 3\cos\frac{1}{2}x}$$

$$y''+7y'+10y=0$$

$$y''+7y'+10y = 4ue^{-2t} - 4u'e^{-2t} + u''e^{-2t}$$
$$-14ue^{-2t} + 7u'e^{-2t}$$
$$+10ue^{-2t}$$

$$= u''e^{-2t} + 3u'e^{-2t} = 0$$

$$e^{-2t}(u''+3u')=0$$

Next page

$y=e^{-2t}$ is one sol

Assume $\boxed{y=ue^{-2t}}$ is a second solution.

$$y' = -2ue^{-2t} + u'e^{-2t}$$

$$y'' = 4ue^{-2t} - 2u'e^{-2t} - 2u'e^{-2t} + u''e^{-2t}$$

$$y'' = 4ue^{-2t} - 4u'e^{-2t} + u''e^{-2t}$$

$u = u(t)$

$u'' + 3u' = 0$ Let $v = v(t) = u'$

$v' + 3v = 0$

$v = u'$

$v' = u''$

$\frac{dv}{dt} = -3v$

$v = e^{-3t}$

$u' = e^{-3t}$

$u = -\frac{1}{3} e^{-3t} + c$

Ignore

$y_1 = e^{-2t}$

$y_2 = u e^{-2t}$

$y_2 = e^{-5t}$

Gen sol: $y = c_1 e^{-2t} + c_2 e^{-5t}$