Solve $y'' + 5y' + 4y = 0$

$y'' + 5y' + 4y = 5\sin 3t, \; y(0) = a, \; y'(0) = b$

$$y'' + 5y' + 4y = 0$$

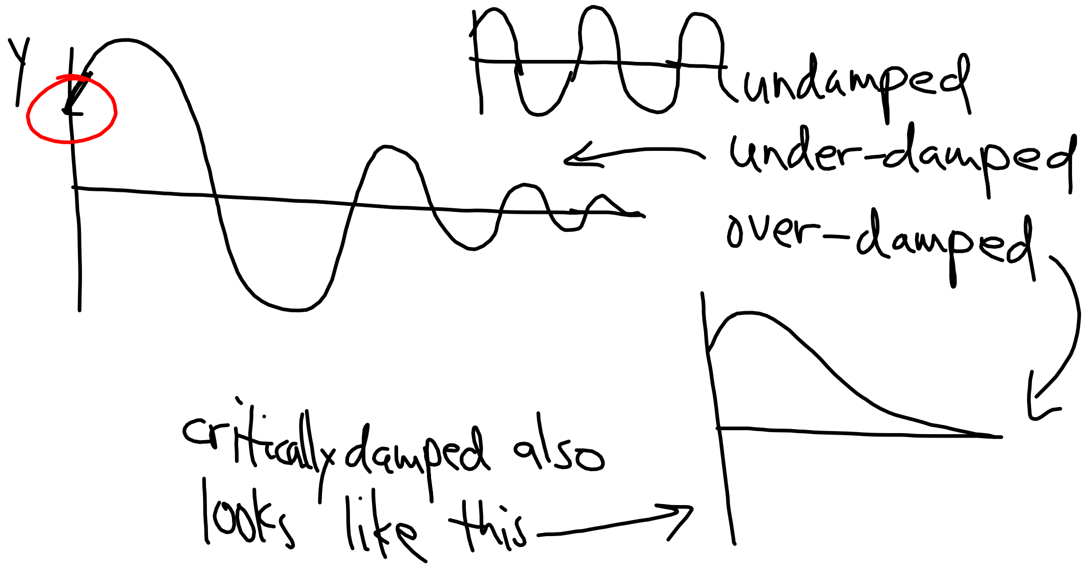
$y = e^{rt}$
guess for sol

$$r^2 + 5r + 4 = 0$$

$$(r+1)(r+4) = 0$$

$$r = -1, -4 \longrightarrow \boxed{y = C_1 e^{-t} + C_2 e^{-4t}}$$

ODE	Solution	b	b^2-4ac
undamped $y''+25y=0$	$y=C_1\sin 5t+C_2\cos 5t$	0	<0
under-damped $5y''+6y'+80y=0$	$y=e^{-0.6t}(C_1\sin 4t+C_2\cos 4t)$	>0	<0
over-damped $y''+5y'+4y=0$	$y=C_1e^{-t}+C_2e^{-4t}$	>0	>0
critically damped $y''+4y'+4y=0$	$y=C_1e^{-2t}+C_2te^{-2t}$	>0	$=0$


y
undamped
under-damped
over-damped
critically damped also
looks like this

$$y'' + 9y = \underbrace{5e^{-2t}}$$

$\boxed{\text{Solve}}$

Guess $y = Ae^{-2t}$ and make it "work!"

$y' = -2Ae^{-2t}$

$y'' = 4Ae^{-2t}$

$$4Ae^{-2t} + 9Ae^{-2t} = 5e^{-2t}$$

$$13Ae^{-2t} = 5e^{-2t}$$

$$13A = 5$$

$$A = \frac{5}{13}$$

Particular Solution

$$\boxed{y = \frac{5}{13}e^{-2t}}$$

General Solution: $y = C_1 \sin 3t + C_2 \cos 3t + \frac{5}{13}e^{-2t}$

$y'' + 7y' + 10y = 5t^2 - 8$ Ex. 3.4(b)

$\boxed{\text{Guess} \quad y = At^2 + Bt + C}$

$2A + 7(2At + B) + 10(At^2 + Bt + C) = 5t^2 + 0t - 8$

$2A + \cancel{14At} + 7B + \cancel{10At^2} + \cancel{10Bt} + 10C = 5t^2 + 0t - 8$

$\underbrace{10A}_{5}t^2 + \underbrace{(14A + 10B)}_{0}t + \underbrace{(2A + 7B + 10C)}_{-8} = 5t^2 + 0t - 8$

$10A = 5 \qquad 14A + 10B = 0 \qquad 2A + 7B + 10C = -8$

$A = \frac{1}{2} \rightsquigarrow \quad 7 + 10B = 0$

$B = -\frac{7}{10}$