① Solve $y''+6y'+5y=0$.

② Find values of A and B for which $y=A\sin 2t+B\cos 2t$ is a solution to $y''+6y'+5y=5\cos 2t$

Solve $y''+6y'+5y=5\cos 2t$, $y(0)=1$, $y'(0)=2$

1) Solve $y''+6y'+5y=0$

$$r^2+6r+5=0$$

$$(r+1)(r+5)=0$$

$$r=-1,-5$$

$$y_h=C_1e^{-t}+C_2e^{-5t}$$ Homogeneous Solution

② Find a particular solution to

$$y'' + 5y' + 6y = 5\cos 2t$$

Guess $y = A\sin 2t + B\cos 2t$

⋮ some small of pain...

$A = \frac{12}{29}$, $B = \frac{1}{29}$

$y_p = \frac{12}{29}\sin 2t + \frac{1}{29}\cos 2t$ Particular Solution

③ The general sol to $y''+6y'+5y=5\cos 2t$

omit

$y = y_h + y_p = C_1e^{-t} + C_2e^{-5t} + \frac{12}{29}\sin 2t + \frac{1}{29}\cos 2t$

④ Use initial conditions $y(0)=1$, $y'(0)=2$ to find C_1 and C_2.

$1 = C_1 + C_2 + \frac{1}{29}$ $\quad 2 = -C_1 - 5C_2 + \frac{24}{29}$

$y' = -C_1e^{-t} - 5C_2e^{-5t} + \frac{24}{29}\cos 2t - \frac{2}{29}\sin 2t$

$1 = C_1 + C_2 + \frac{1}{29}$

$2 = -C_1 - 5C_2 + \frac{24}{29}$

$3 = -4C_2 + \frac{25}{29}$

$4C_2 = \frac{25}{29} - \frac{87}{29}$

$\frac{1}{4} \cdot 4C_2 = -\frac{\cancel{62}^{31}}{29} \cdot \frac{1}{\cancel{4}_2}$

$\boxed{C_2 = -\frac{31}{58}}$

$\begin{array}{r} 87 \\ -25 \\ \hline 62 \end{array}$

$1 = C_1 - \frac{31}{58} + \frac{2}{58}$

$\frac{58}{58} = C_1 - \frac{29}{58}$

$C_1 = \frac{87}{58} = \frac{3}{2}$

$$\boxed{y = \frac{3}{2}e^{-t} - \frac{31}{58}e^{-5t} + \frac{12}{29}\sin 2t + \frac{1}{29}\cos 2t}$$