Solve $\frac{dT}{dt} = k(T-73)$ by separation of variables, assuming $T > 73$.

$$dT = k(T-73)\,dt$$

$$\frac{dT}{T-73} = k\,dt$$

$$\ln|T-73| = kt + C$$

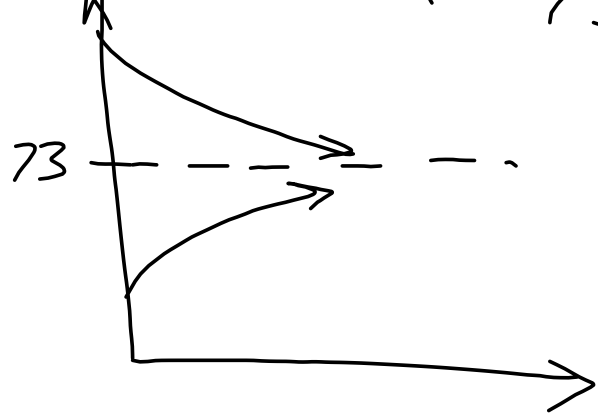
$$e^{\ln(T-73)} = e^{kt+C}$$

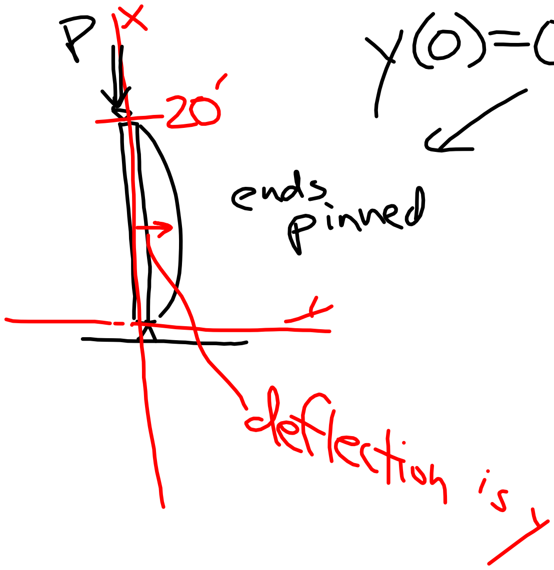
$$= e^{kt}e^{C}$$

$$T-73 = Ce^{kt}$$

$$T = 73 + Ce^{kt}$$

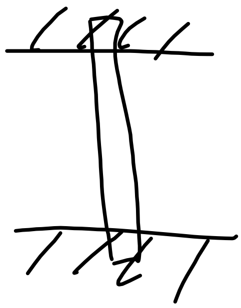
$k<0$





$y(0)=0$, $y(20)=0$

ends pinned



$y'(0)=0$

$y'(20)=0$

$$EI\frac{d^2y}{dx^2} = -Py \Longrightarrow \frac{d^2y}{dx^2} = -\frac{P}{EI}y$$

E: modulus of elasticity

I: cross-sectional moment of inertia

$$\frac{d^2}{dx^2}(y) = -\frac{P}{EI}y$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI}y = 0 \qquad Ay = \lambda y$$

$$\frac{d^2y}{dx^2} + \left(\sqrt{\frac{P}{EI}}\right)^2 y = 0$$

$$\frac{d^2y}{dx^2} + \lambda^2 y = 0$$

$$y = C_1 \sin\lambda x + C_2 \cos\lambda x$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI}y = 0$$

$$r^2 + \frac{P}{EI} = 0$$

$$r^2 = -\frac{P}{EI}$$

$$r = \pm\sqrt{\frac{P}{EI}}\,i$$

$$\boxed{y = C_1 \sin\sqrt{\frac{P}{EI}}x + C_2 \cos\sqrt{\frac{P}{EI}}x}$$

$$\boxed{y = C_1 \sin\sqrt{\frac{P}{EI}}x + C_2 \cos\sqrt{\frac{P}{EI}}x}$$

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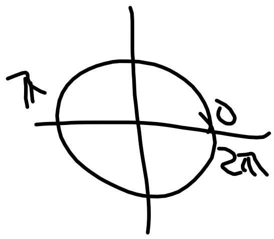
$y(0) = 0 \Rightarrow C_2 = 0$

$y(20) = 0$

$$y = C \sin\sqrt{\frac{P}{EI}}x$$

$E = 800, \ I = 150$

$$0 = C \sin\underbrace{\sqrt{\frac{P}{EI}}(20)}$$



$$\sqrt{\frac{P}{EI}}(20) = \pi, 2\pi, 3\pi, 4\pi, \ldots$$

$$\sqrt{\frac{P}{EI}} = \frac{\pi}{20}, \frac{2\pi}{20}, \frac{3\pi}{20}, \frac{4\pi}{20}, \ldots$$

Solutions:

buckling modes

$$y = C \sin\frac{\pi}{20}x, \ C \sin\frac{2\pi}{20}x, \ C \sin\frac{3\pi}{20}x, \ldots$$

$$\sqrt{\frac{P}{EI}} = \frac{\pi}{20}, \frac{2\pi}{20}, \frac{3\pi}{20}, \frac{4\pi}{20}, \ldots$$

What does this tell us about P?

$$\frac{P}{EI} = \frac{\pi^2}{400}, \frac{4\pi^2}{400}, \frac{9\pi^2}{400}, \frac{16\pi^2}{400}$$

E=800
I=150
EI=120,000

$$\frac{P}{120{,}000} = \quad ''$$

$$= \cancel{120{,}000}^{300}\left(\frac{\pi^2}{\cancel{400}}\right), 120{,}000\left(\frac{4\pi^2}{400}\right)$$

$$\frac{120{,}0\cancel{00}}{4\cancel{00}}$$

Critical loads

$$= 1(300\pi^2), 4(300\pi^2), 9(300\pi^2), \ldots$$