Solve $y'' + \frac{P}{10{,}000}y = 0$, $y'(0)=0, y'(20)=0$

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$r^2 + \frac{P}{10000} = 0$

$r^2 = -\frac{P}{10000}$

$r = \pm\sqrt{\frac{P}{10000}}\,i$

$r = \pm\frac{\sqrt{P}}{100}\,i$

$y = C_1 \sin\frac{\sqrt{P}}{100}x + C_2\cos\frac{\sqrt{P}}{100}x$

$y'(0)=0, y'(20)=0$

$y' = C_1\frac{\sqrt{P}}{100}\cos\frac{\sqrt{P}}{100}x - C_2\frac{\sqrt{P}}{100}\sin\frac{\sqrt{P}}{100}x$

$0 = C_1\frac{\sqrt{P}}{100}$ (From $y'(0)=0$)

$C_1 = 0$, so

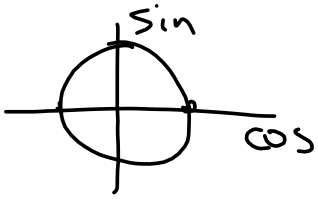
$y = C\cos\frac{\sqrt{P}}{100}x$

$y' = -C\frac{\sqrt{P}}{100}\sin\frac{\sqrt{P}}{100}x$

$y = C\cos\frac{\sqrt{P}}{100}x$

$y' = -C\frac{\sqrt{P}}{100}\sin\frac{\sqrt{P}}{100}x$

$y'(20) = 0$



$0 = -C\frac{\sqrt{P}}{100}\sin\frac{\sqrt{P}}{100}(20)$

must be zero

$\frac{\sqrt{P}}{100}(20) = \pi, 2\pi, 3\pi, 4\pi, \ldots$

$\frac{\sqrt{P}}{100} = \frac{\pi}{20}, \frac{2\pi}{20}, \frac{3\pi}{20}, \frac{4\pi}{20}, \ldots$

Solutions: $y = C\cos\frac{\pi}{20}x, C\cos\frac{2\pi}{20}x, C\cos\frac{3\pi}{20}x, \ldots$

buckling modes

$\sqrt{P} = 5\pi, 2(5\pi), 3(5\pi), 4(5\pi), \ldots$

$P = 25\pi^2, 4(25\pi^2), 9(25\pi^2), 16(25\pi^2), \ldots$

$(3\cdot 2)^2 = 3\cdot 2\cdot 3\cdot 2$

1st critical load

2nd critical load

3rd C.L.

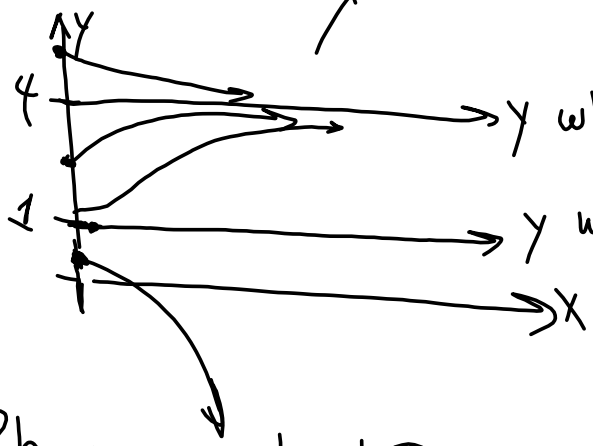
$$y'' + \frac{P}{10{,}000}y = 0 \qquad y(0)=0,\ y(20)=0$$

$$y = C\sin\frac{\pi}{20}x,\ C\sin\frac{2\pi}{20}x,\ C\sin\frac{2\pi}{20}x,\ \ldots$$

$\frac{dy}{dx} = -(y-1)(y-4)$ $\frac{dy}{dx} = F(x,y)$

x, y

What is y?



unstable equilibrium at $y=1$

stable equilibrium $y=4$

Phase portrait?