① One solution to $y'' + 2y' + y = 0$ is $y = e^{-t}$.
Assume another solution of the form $y = ue^{-t}$, where u is a function of t, and find this second solution.

$r^2 + 2r + 1 = 0$
$(r+1)(r+1) = 0$
$r = -1$
$y = e^{-t}$

$y = e^{-t}$

Yada, yada

$y' =$
$y'' =$

$u''e^{-t} = 0$
$u'' = 0$
$u' = c_1$
$u = c_1 t + c_2$

$y = ue^{-t} = c_1 te^{-t} + c_2 e^{-t}$

$$y'' + \frac{1}{4}y = 0 \qquad y'(0) = 1,\ y'(\pi) = 2$$

unknown

λ, $\sqrt{\frac{P}{10,000}}$

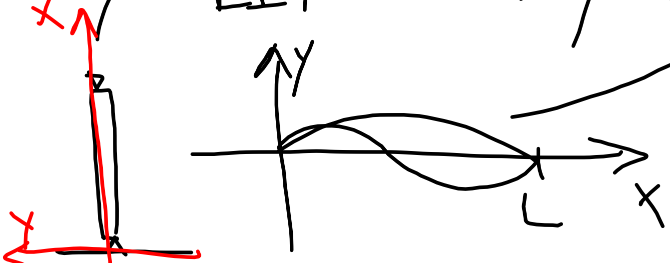
$$r^2 + \frac{1}{4} = 0$$

$$r^2 = -\frac{1}{4}$$

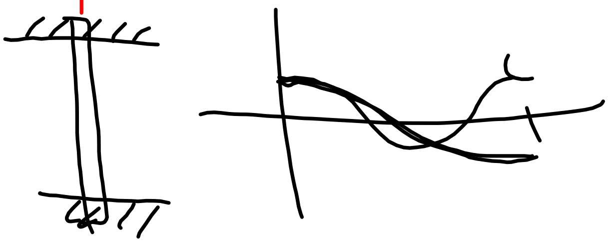
$$r = \pm\frac{1}{2}i$$

$$y = C_1 \sin\tfrac{1}{2}x + C_2 \cos\tfrac{1}{2}x$$

$$y'' + \frac{P}{EI}y = 0 \Longrightarrow y = C_1 \sin\sqrt{\frac{P}{EI}}\,x + C_2 \cos\sqrt{\frac{P}{EI}}\,x$$

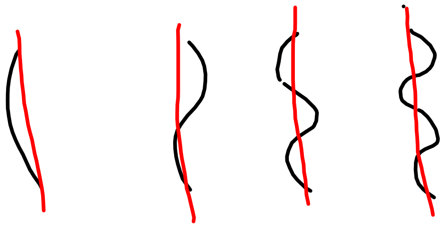


$$y(0)=0,\ y(L)=0$$



$$y'(0)=0,\ y'(L)=0$$

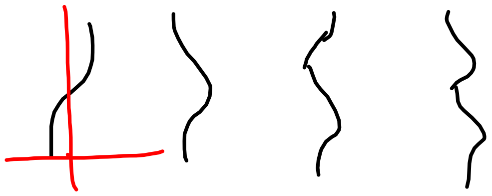
Pinned-Pinned $\longrightarrow y(0)=0, y(L)=0$



1st 2nd 3rd 4th

Buckling Modes

Embedded - Embedded $\quad y'(0)=0, \; y'(L)=0$



1st 2nd 3rd 4th $\longrightarrow$ Drift allowed

1st 2nd $\longrightarrow$ No drift

$y(0)=y(L)$

$$x^2y''+2xy'-6y=0$$

$y=x^2$ is a sol

$y=ux^2$

$y'=2xu+u'x^2$

$y''=2xu'+2u+2xu'+u''x^2$

$=u''x^2+4xu'+2u$

$$x^2y''+2xy'-6y=u''x^4+4x^3u'+2ux^2$$
$$+2x^3u'+4x^2u$$
$$-6ux^2$$
$$=u''x^4+6u'x^3=0$$

$$x^3(u''x+6u')=0$$

$$u''x+6u'=0$$

$v=u'$

$v'=u''$

$$v'x+6v=0$$

$$x\frac{dv}{dx}=-6v$$

$$\frac{dv}{v}=-6\frac{dx}{x}$$

$$\ln v=-6\ln x$$

$$\ln v=\ln x^{-6}$$

$$v=x^{-6}$$

$$u'=x^{-6}$$

$$u=Cx^{-5}$$

Second sol is $y=ux^2=x^{-5}x^2=x^{-3}$