Math 321 Formula Sheet

Exponents and Logarithms:

$$x^a x^b = x^{a+b}, \qquad (x^a)^b = x^{ab}, \qquad x^{-n} = \frac{1}{x^n}, \qquad x^{\frac{1}{n}} = \sqrt[n]{x}$$

$$\log_a(uw) = \log_a u + \log_a w \qquad \log_a\left(\frac{u}{w}\right) = \log_a u - \log_a w \qquad \log(u^c) = c\log u$$

$$e^{\ln u} = u, \qquad \ln e^u = u$$

Trigonometric Functions:

$$\tan u = \frac{\sin u}{\cos u} \qquad \cot u = \frac{\cos u}{\sin u} \qquad \sec u = \frac{1}{\cos u} \qquad \csc u = \frac{1}{\sin u}$$

Trigonometric Identities:

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

$$\sin 2\theta = 2\sin\theta\cos\theta \qquad \cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

Useful Trigonometric Factoid: $A\sin\omega t + B\cos\omega t = C\sin(\omega t + \phi)$, where

$$C = \sqrt{A^2 + B^2} \text{ and } \phi = \tan^{-1}\frac{B}{A} \text{ if } A > 0, \quad \phi = \pi + \tan^{-1}\frac{B}{A} \text{ if } A < 0$$

$$\text{If } A = 0, \text{ then } B\cos\omega t = B\sin\left(\omega t + \tfrac{\pi}{2}\right)$$

Euler's Relations: $e^{i\theta} = \cos\theta + i\sin\theta \qquad e^{-i\theta} = \cos\theta - i\sin\theta$

Product Rule: $[uv]' = uv' + vu'$ or $\dfrac{d[uv]}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

Derivatives of Trig, Exponential and Log Functions:

$$[\sin(u)]' = [\cos(u)](u)', \quad (\sin at)' = a\cos at \qquad [\cos(u)]' = [-\sin(u)](u)', \quad (\cos at)' = -a\sin at$$

$$[e^u]' = [e^u](u)', \quad (e^{at})' = ae^{at} \qquad [\ln(u)]' = \frac{1}{u}(u)'$$

A Few Integration Formulas: All formulas should include an arbitrary constant, which I have left off here to keep things a little cleaner.

$$\int dx = x \qquad \int cf(x)\,dx = c\int f(x)\,dx \qquad \int k\,dx = kx$$

$$\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$

$$\int u^n\,du = \frac{1}{n+1}\,u^{n+1} \text{ as long as } n \neq -1 \qquad \int u^{-1}\,du = \int \frac{1}{u}\,du = \ln|u|$$

$$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln|\,ax+b\,| \qquad \int e^u\,du = e^u \qquad \int e^{at}\,dt = \frac{1}{a}\,e^{at}$$

$$\int t\,e^{at}\,dt = \frac{e^{at}(at-1)}{a^2} \qquad \int t^2e^{at}\,dt = \frac{e^{at}(a^2t^2-2at+2)}{a^3}$$

$$\int \sin u\,du = -\cos u \qquad \int \sin at\,dt = -\frac{1}{a}\cos at$$

$$\int \cos u\,du = \sin u \qquad \int \cos at\,dt = \frac{1}{a}\sin at$$

$$\int e^{at}\sin bt\,dt = \frac{e^{at}}{a^2+b^2}\,(a\sin bt - b\cos bt) \qquad \int e^{at}\cos bt\,dt = \frac{e^{at}}{a^2+b^2}\,(a\cos bt + b\sin bt)$$

Solving $\dfrac{dy}{dx} + p(x)y = q(x)$ **Using an Integrating Factor:**

- Compute $u = \displaystyle\int p(x)\,dx$, multiply both sides of the equation by e^u
- The left side becomes $\dfrac{d[e^u y]}{dx}$. Multiply both sides by dx and integrate.