Math 321 EXAM 1, FALL 2015 Name: 52 points

Each numbered exercise is worth six points unless stated otherwise.

1. Solve the initial value problem $\frac{dy}{dx} - y^2 = 5y^2e^{3x}$, $y(0) = 3$ in the space below using separation of variables. **Show all steps clearly in the space below and DO NOT solve for** y. *9 points*

Solution: $-\frac{1}{y} = \frac{5}{3}e^{3x} + x - 2$

$$\frac{dy}{dx} = 5y^2e^{3x} + y^2$$

$$\frac{dy}{dx} = y^2(5e^{3x}+1)$$

$$y^{-2}dy = (5e^{3x}+1)dx$$

$$-\frac{1}{y} = \frac{5}{3}e^{3x} + x + C$$

$$-\frac{1}{3} = \frac{5}{3} + C$$

$$C = -2$$

Separate $+1\frac{1}{2}$
$\int$ each side $+1\frac{1}{2}$ each
$+C$ in sol $+1\frac{1}{2}$] 6

Find C idea $+1$
finding $+2$] 3

2. Use an integrating factor to solve the ODE $\frac{dy}{dt} + 2y = 3t$. **Show all steps**, and consider using the formula sheet...

$u = \int 2dt = 2t$ $+1$

$$e^{2t}\frac{dy}{dt} + 2ye^{2t} = 3te^{2t} \quad +\tfrac{1}{2}$$

$$\frac{d(ye^{2t})}{dt} = 3te^{2t} \quad +\tfrac{1}{2}$$

$$\int d(ye^{2t}) = \int 3te^{2t}dt \quad +\tfrac{1}{2}$$

$$ye^{2t} = \frac{3e^{2t}(2t-1)}{4} + C \quad +1\tfrac{1}{2} \quad +1$$

$$\boxed{y = \frac{3}{4}(2t-1) + Ce^{-2t}} \quad +1$$

Course

9	6 4
8	0 4 3 0 8 0
7	6 8 0 0 1 8 7 8
6	1 3 8
5	9 8
4	8

+15

Do **ALL BUT ONE of the remaining exercises.** Put an X through the one that you *DO NOT* want me to grade.

3. Determine values of the constants A and B for which $y = A\sin t + B\cos t$ is a solution to the ODE $y'' - 3y' - 4y = 2\sin t$. **Show clearly how you do this, and give your answers in exact form.**

$y' = A\cos t - B\sin t \quad +\frac{1}{2}$

$y'' = -A\sin t - B\cos t \quad +\frac{1}{2}$

+1 for putting in

$y'' - 3y' - 4y = -A\sin t - B\cos t - 3(A\cos t - B\sin t) - 4(A\sin t + B\cos t)$

$= (-5A + 3B)\sin t + (-3A - 5B)\cos t$

x1 correct

x1 idea

x2

$= 2\sin t$

$\boxed{\begin{matrix} -5A + 3B = 2 \\ -3A - 5B = 0 \end{matrix}}$ $\Longrightarrow$ $-25A + 15B = 10$, $-9A - 15B = 0$; $-34A = 10 \longrightarrow \boxed{A = -\frac{5}{17}}$

+1 each

$-5A + 3B = 2 \Longrightarrow -15A + 9B = 6$

$-3A - 5B = 0 \Longrightarrow 15A + 25B = 0$

$34B = 6 \longrightarrow \boxed{B = \frac{3}{17}}$

$\frac{dy}{dx} = \frac{\cos x - xy}{y}$

4. Use the following ODEs for this exercise. *2 points each part*

I. $y'' - 5xy' = e^x + 1$ II. $y\frac{dy}{dx} + xy = \cos x$ III. $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} - 15x = 0$ IV. $\frac{dy}{dx} - 5y = y^2$

(a) Recall that a linear ODE is one that can be written in the form

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1}y}{dx^{n-1}} + \cdots + a_2(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_0(x)y = f(x)$$

Give the Roman numerals of all of the above that are linear: I, III

(b) Give the Roman numerals of all that are separable: IV

(c) Give the Roman numerals of all that are homogeneous: III

5. A mass is hanging on a spring in an oil bath, as shown below and to the right. The mass is set in motion by pulling it down 3 centimeters and then giving it a push downward at 2 centimeters per second. The oil bath causes damping, so the vibration of the mass on the spring will eventually die out. Identify each of the following as an independent variable (IV), dependent variable (DV), parameter (P), initial condition (IC) or boundary condition (BC).

P the mass m of the mass

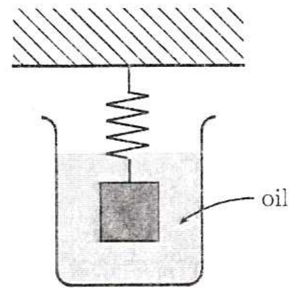
IV time

IC pulling the mass down 3 centimeters

P the viscosity of the oil

IC pushing the mass downward at 2 centimeters per second

DV the height of the mass at any time

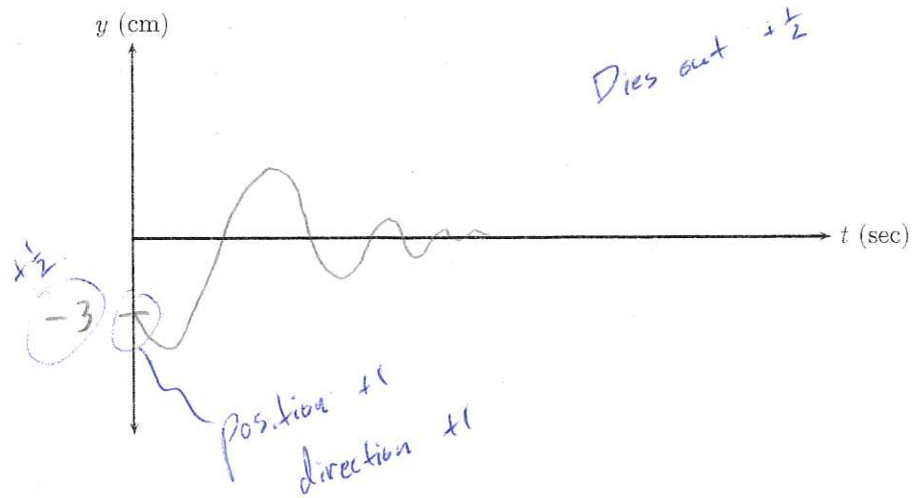


6. Consider again the situation from the previous exercise. Let y be the height of the mass in centimeters and t be time in seconds.

(a) Take time zero to be when the mass is set in motion. In the space below and to the left, give the information that the mass is set in motion by pulling it down 3 centimeters and then giving it a push downward at 2 centimeters per second using function notation.

$y(0) = -3$

$y'(0) = -2$



(b) Sketch a graph of the height of the mass versus time on the axes above and to the right. **You should be able to label one non-zero numerical value on one axis - do so.**

7. Is $y = -\frac{3}{2}e^{-x^2} + 1$ a solution to the initial value problem $\frac{dy}{dx} + 2xy = x$, $y(0) = -\frac{1}{2}$?

Circle one: yes no **Show work supporting your answer in the space below.**

$y(0) = -\frac{3}{2}e^{0} + 1 = -\frac{1}{2}$ +2

$\frac{dy}{dx} = 3xe^{-x^2}$ +2

Conclusion +2

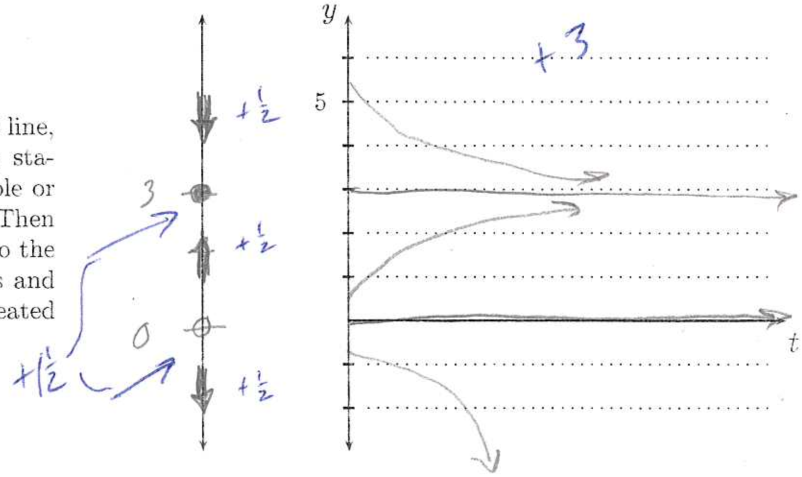
$\frac{dy}{dx} + 2xy = 3xe^{-x^2} + 2x\left(-\frac{3}{2}e^{-x^2}+1\right) = 3xe^{-x^2} - 3xe^{-x^2} + 2x = 2x \neq x$

+2

8. Consider the autonomous ODE

$$\frac{dy}{dx} = y(3-y).$$

Sketch the phase diagram on the vertical line, labelling critical points and indicating stable equilibria with solid dots and unstable or semi-stable equilibria with open circles. Then graph some solution curves on the grid to the right, including all equilibrium solutions and at least one solution in each interval created by the critical values.



9. At some time we'll call zero you are 210 miles from Klamath Falls, *driving away from here* at a constant speed of 63 mph. Let x represent your distance from Klamath Falls and t the time after time zero.

(a) **Write two mathematical statements** representing the information given. *4 points*

$\boxed{\frac{dx}{dt} = 63, \quad x(0) = 210}$

(b) Give an equation for the distance x from Klamath Falls as a function of the time t. *2 points*

$\boxed{x = 63t + 210}$