Math 321 January 16 Class Winter 2018

The following statement and two questions are taken from a study guide to the *Fundamentals of Engineering (FE)* exam. **Answer the questions BELOW the box**, not the ones in it.

> Under certain conditions the motion of an oscillating spring is described by the differential equation $\frac{d^2x}{dt^2} + 16x = 0$, where x is the displacement in feet of the end of the spring, and t is the time in seconds. At $t = 0$ seconds, the displacement is $\frac{1}{4}$ foot upward and the velocity is 0 feet per second; that is, $x(0) = \frac{1}{4}$ and $x'(0) = 0$.
>
> 1. What is the general solution of the system? (c_1 and c_2 are constants.)
>
>   (a) $x = c_1 e^{-t} + c_2 e^{-3t}$ (b) $x = c_1 e^{-4t} + c_2 e^{4t}$ (c) $x = c_1 \sin 4t$
>
>   (d) $x = c_1 \cos 4t$ (e) $x = c_1 \cos 4t + c_2 \sin 4t$
>
> 2. The solution that fits the initial conditions is
>
>   (a) $x = \frac{1}{4} e^{-4t}$ (b) $x = \frac{1}{3} \sin 4t$ (c) $x = 4 \cos 4t$
>
>   (d) $x = \frac{1}{4} \cos 4t$ (e) $x = \frac{1}{4} \cos 4t + \frac{1}{3} \sin 4t$

1. *Without doing any computations* you can determine that two of the choices cannot be the answer to **of Exercise 1**. Which two are they, and why?

2. Determine whether choice (b) **of Exercise 1** is a solution to the equation.

3. Determine which of the choices **for Exercise 2** satisfies the initial conditions.

4. Is $y = 2e^{-3t} - \frac{1}{13} \sin 2t - \frac{8}{13} \cos 2t$ a solution to the initial value problem

$$y'' + 4y' + 3y = 5 \sin 2t, \qquad y(0) = \tfrac{5}{13}, \ \ y'(0) = -\tfrac{67}{13}?$$

Equation 1: $\dfrac{dy}{dx} + 3y = 0$

Equation 2: $x^2y'' + xy' + x^2y = 0$

Equation 3: $y'' + 9y = 13e^{-2t}$

Equation 4: $15.3\dfrac{d^4y}{dx^4} = 1.4$

Equation 5: $\dfrac{dy}{dx} = \dfrac{x}{y}$

Equation 6: $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = \dfrac{\partial u}{\partial t}$

$$a_n(x)\frac{d^ny}{dx^n} + a_{n-1}(x)\frac{d^{n-1}y}{dx^{n-1}} + \cdots + a_2(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_0(x)y = f(x)$$

A. $\dfrac{dy}{dx} - \dfrac{x}{y} = 0$

B. $y' - 2xy = x$

C. $y' + 2y = y^2$

D. $5\dfrac{dy}{dx} - 3y = \sin x$

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1}y}{dx^{n-1}} + \cdots + a_2(x)\frac{d^2 y}{dx^2} + a_1(x)\frac{dy}{dx} + a_0(x)y = f(x)$$

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Autonomous: $\dfrac{dy}{dx} = F(y)$

Separable: $\dfrac{dy}{dx} = g(x)h(y)$