Math 321 SOLVING SECOND ORDER IVPS **Winter 2018**

This handout is to help you learn the process for solving a second order IVP of the form

$$ay'' + by' + cy = f(t), \quad y(0) = y_0,\ y'(0) = y_0'.$$

Here a, b, c, y_0 and y_0' are all constants. (y_0 and $y'(0)$ are the given initial values.) The process basically consists of three parts - more detail about each will be given below.

1) Solve the homogeneous equation $ay'' + by' + cy = 0$ to find the homogeneous solution y_h. It will contain two unknown constants C_1 and C_2 *that you will find later*.
2) Use the method of undetermined coefficients to find the particular solution y_p of $ay'' + by' + cy = f(t)$.
3) Add the homogeneous and particular solutions to get the general solution $y = y_h + y_p$, which will contain the two unknown constants C_1 and C_2. Apply the initial conditions to determine their values, then give the final solution.

Find the Homogeneous Solution

- Form the homogeneous ODE $ay'' + by' + cy = 0$.
- Solve the auxiliary equation $ar^2 + br + c = 0$ and solve to obtain one or (usually) two values for r.
- Give the homogeneous solution, which will have one of four forms. Here they are for specific values of r:

  - $r = -2, -5$: $y_h = C_1 e^{-2t} + C_2 e^{-5t}$
  - $r = -3$: $y_h = C_1 e^{-3t} + C_2 t e^{-3t}$
  - $r = \pm 3i$: $y_h = C_1 \sin 3t + C_2 \cos 3t$
  - $r = -2 \pm 3i$: $y_h = e^{-2t}(C_1 \sin 3t + C_2 \cos 3t)$

Find the Particular Solution

Force the appropriate guess for the particular solution y_p to "work" in the differential equation $ay''+by'+cy = f(t)$. Here are the appropriate guesses for y_p for specific examples of the the three forms of $f(t)$ that you will encounter:

- If $f(t) = 3e^{-5t}$, guess $y_p = Ae^{-5t}$.
- If $f(t) = 2\cos 3t$, guess $y_p = A\sin 3t + B\cos 3t$. **Note that your guess must have both a sine and cosine even if $f(t)$ just has one of them**.
- If $f(t) = 5x^2 + 7$, guess $y_p = At^2 + Bt + C$. When $f(t)$ is a degree n polynomial, the guess for y_p must be a degree n polynomial with **all terms present**, each with an unknown coefficient.

After substituting the guess into the ODE, you must group all like terms (sines or cosines, powers of t) together on the left side and equate their coefficients with the coefficients from $f(t)$. This is called the **method of undetermined coefficients**.

Finding the General Solution to the Initial Value Problem

The general solution to the ODE $ay'' + by' + cy = f(t)$ is $y = y_h + y_p$, and it will contain the two unknown constants from y_h. Find the derivative y' and apply the two initial conditions $y(0) = y_0$ and $y'(0) = y_0'$ to find the constants C_1 and C_2, as we have done before.

Conclude by giving the final solution, with no unknown constants.