Math 321 SECOND ORDER IVP PRACTICE **Winter 2018**

The process for solving a second order IVP of the form

$$ay'' + by' + cy = f(t), \quad y(0) = \alpha,\ y'(0) = \beta$$

is the following:

1) Solve the homogeneous equation $ay'' + by' + cy = 0$ to find the homogeneous solution y_h. It will contain two unknown constants *that you will find later*.

2) Use the method of undetermined coefficients to find the particular solution y_p of $ay'' + by' + cy = f(t)$.

3) Add the homogeneous and particular solutions to get the general solution $y = y_h + y_p$, which will contain two unknown constants. Apply the initial conditions to determine the constants.

4) End by giving the final solution!

Here are four practice exercises - the solutions are given at the bottom of the page. The Symbolab tool at

https://www.symbolab.com/solver/ordinary-differential-equation-calculator

solves IVPs and shows the steps of the solution. Work all in exact form except Exercise 4, which you may do in decimal form. Round as you feel appropriate, and check your answer by converting the give answer to decimal form.

1. $y'' + 3y' + 2y = 5\sin 2t, \quad y(0) = -1,\ y'(0) = 2$

2. $y'' - y' - 2y = 4t^2, \quad y(0) = 3,\ y'(0) = 2$

3. $y'' + 3y' + 2y = 3\cos 2t, \quad y(0) = 1,\ y'(0) = 0$

4. $y'' + 6y' + 11y = 7e^{-5t}, \quad y(0) = 0,\ y'(0) = 1$

Solutions:

1. $y = 2e^{-t} - \frac{9}{4}e^{-2t} - \frac{1}{4}\sin 2t - \frac{3}{4}\cos 2t$

2. $y = 4e^{-t} + 2e^{2t} - 2t^2 + 2t - 3$

3. $y = \frac{7}{5}e^{-t} - \frac{1}{4}e^{-2t} + \frac{9}{20}\sin 2t - \frac{3}{20}\cos 2t$

4. $y = e^{-3t}\left(\frac{5\sqrt{2}}{3}\sin\sqrt{2}t - \frac{7}{6}\cos\sqrt{2}t\right) + \frac{7}{6}e^{-5t}$