

Math 321, Assignment 19 5.3:2

$$\frac{d^2 y}{dx^2} + \frac{P}{100,000} y = 0 \quad y(0) = 0, \quad y(12) = 0$$

$$y = C_1 \sin \sqrt{\frac{P}{100,000}} x + C_2 \cos \sqrt{\frac{P}{100,000}} x$$

$$y(0) = 0: 0 = C_1(0) + C_2(1) \implies C_2 = 0$$

$$y = C_1 \sin \sqrt{\frac{P}{100,000}} x$$

$$y(12) = 0: 0 = C_1 \sin \sqrt{\frac{P}{100,000}} (12)$$

$$\sqrt{\frac{P}{100,000}} (12) = 0, \pi, 2\pi, 3\pi, 4\pi, \dots$$

$$\sqrt{\frac{P}{100,000}} = 0, \frac{\pi}{12}, \frac{2\pi}{12}, \frac{3\pi}{12}, \frac{4\pi}{12}, \dots$$

The first four nonzero buckling modes are

$$y = C \sin \frac{\pi}{12} x, C \sin \frac{2\pi}{12} x, C \sin \frac{3\pi}{12} x, C \sin \frac{4\pi}{12} x$$

$$\frac{P}{100,000} = 0, \frac{\pi^2}{144}, \frac{4\pi^2}{144}, \frac{9\pi^2}{144}, \frac{16\pi^2}{144}$$

$$P = 0, \frac{100,000\pi^2}{144}, 4\left(\frac{100,000\pi^2}{144}\right), 9\left(\frac{100,000\pi^2}{144}\right), 16\left(\frac{100,000\pi^2}{144}\right)$$

First four nonzero critical loads

The third nonzero critical load is nine times stronger than the first nonzero critical load.