(3) a) $\frac{d^2y}{dx^2} + \frac{P}{100,000}y = 0 \quad y(0)=0,\ y(6)=0$

$$y = C_1 \sin\sqrt{\frac{P}{100,000}}\,x + C_2 \cos\sqrt{\frac{P}{100,000}}\,x$$

$y(0)=0$: $0 = C_1(0) + C_2(1) \Longrightarrow C_2 = 0$

$$y = C_1 \sin\sqrt{\frac{P}{100,000}}\,x$$

$y(6)=0$: $0 = C_1 \sin\sqrt{\frac{P}{100,000}}\,(6)$

$$\sqrt{\frac{P}{100,000}}\,(6) = 0, \pi, 2\pi, 3\pi, 4\pi, \ldots$$

$$\sqrt{\frac{P}{100,000}} = 0, \frac{\pi}{6}, \frac{2\pi}{6}, \frac{3\pi}{6}, \frac{4\pi}{6}, \ldots$$

Buckling modes:

$$y = C\sin\frac{\pi}{6}x, C\sin\frac{2\pi}{6}x, C\sin\frac{3\pi}{6}x, C\sin\frac{4\pi}{6}x$$

$$\frac{P}{100,000} = 0, \frac{\pi^2}{36}, \frac{4\pi^2}{36}, \frac{9\pi^2}{36}, \frac{16\pi^2}{36}, \ldots$$

$$P = 0, \underbrace{\frac{100000\pi^2}{36}, 4\left(\frac{100,000\pi^2}{36}\right), 9\left(\frac{100,000\pi^2}{36}\right), 16\left(\frac{100,000\pi^2}{36}\right)}_{\text{Non-zero buckling modes}}$$

b) Each of the critical loads is four times the corresponding critical load for the 12 foot column.

c) The first buckling mode for the 6 foot column is half the second buckling mode for the 12 foot column.

(4) a) $\frac{d^2y}{dx^2} + \frac{P}{100,000} y = 0$, $y'(0) = 0, y'(12) = 0$

$$y = C_1 \sin\sqrt{\frac{P}{100,000}}\,x + C_2 \cos\sqrt{\frac{P}{100,000}}\,x$$

$$y' = C_1\sqrt{\frac{P}{100,000}}\cos\sqrt{\frac{P}{100,000}}\,x - C_2\sqrt{\frac{P}{100,000}}\sin\sqrt{\frac{P}{100,000}}\,x$$

$y'(0) = 0$: $0 = C_1\sqrt{\frac{P}{100,000}}(1) - C_2\sqrt{\frac{P}{100,000}}(0) \Rightarrow C_1 = 0$

$$y = C_2 \cos\sqrt{\frac{P}{100,000}}$$

$y'(12) = 0 \Longrightarrow 0 = -C_2\sqrt{\frac{P}{100,000}}\sin\sqrt{\frac{P}{100,000}}\,(12)$

$\sqrt{\frac{P}{100,000}}(12) = 0, \pi, 2\pi, 3\pi, 4\pi, \ldots$

$\sqrt{\frac{P}{100,000}} = 0, \frac{\pi}{12}, \frac{2\pi}{12}, \frac{3\pi}{12}, \frac{4\pi}{12}, \ldots$

Buckling modes; drift allowed

$y = C\cos\frac{\pi}{12}x, C\cos\frac{2\pi}{12}x, C\cos\frac{3\pi}{12}x, C\cos\frac{4\pi}{12}x, C\cos\frac{5\pi}{12}, C\cos\frac{6\pi}{12}$

Buckling modes, no drift

$\frac{P}{100,000} = 0, \frac{\pi^2}{144}, \frac{4\pi^2}{144}, \frac{9\pi^2}{144}, \frac{16\pi^2}{144}, \frac{25\pi^2}{144}, \ldots$

Critical loads, drift allowed:

$P = 0, \frac{100,000\pi^2}{144}, 4\left(\frac{100,000\pi^2}{144}\right), 9\left(\frac{100,000\pi^2}{144}\right), 16\left(\frac{100,000\pi^2}{144}\right), 25\left(\frac{100,000\pi^2}{144}\right)$

Critical loads with no drift

$36\left(\frac{100,000\pi^2}{144}\right)$

See textbook for answers to parts (c) and (d)